MATH 1201 – College Algebra II and Trig **Name** Key
Exam 1, Sec 3.3-3.5, 9.1, 9.2, 9.5, 10.1-10.3 Blitzer, 4^{th} ed.
Spring 2010 Bordwell **80 pts**

Work neatly and show all algebraic and numerical work. No Calculators allowed on the exam.

1. Use long division to divide $6x^4+5x^3+3x-5$ by $3x^2-2x$. **(6 pts)**

$$\boxed{2x^2+3x+2+\frac{7x-5}{3x^2-2x}}$$

$$3x^2-2x \,\overline{)\, 6x^4+5x^3+0x^2+3x-5}$$
$$-6x^4+4x^3$$
$$9x^3+0x^2$$
$$-9x^3+6x^2$$
$$6x^2+3x$$
$$-6x^2+4x$$
$$7x-5$$

2. Given $f(x)=2x^4-5x^3-x^2+3x+2$, use synthetic division and the Remainder Theorem to find $f\left(-\frac{1}{2}\right)$. **(4 pts)**

$-\frac{1}{2}$	2	-5	-1	3	2
		-1	3	-1	-1
	2	-6	2	2	$\boxed{1}$

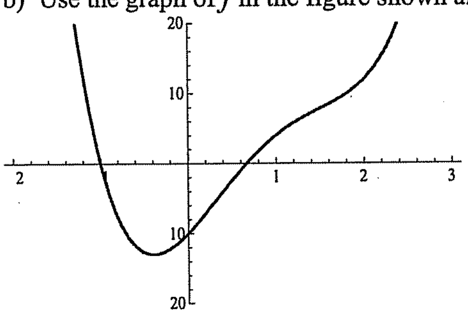
$$\boxed{f(-\tfrac{1}{2})=1}$$

3. Consider the function whose equation is given by; $f(x)=3x^4-11x^3+9x^2+13x-10$

a) List all the possible rational zeros. **(3 pts)**

$$\frac{p}{q}=\frac{\pm 10, \pm 5, \pm 2, \pm 1}{\pm 3, \pm 1}=\boxed{\pm\frac{10}{3}, \pm\frac{5}{3}, \pm\frac{2}{3}, \pm\frac{1}{3}, \pm 10, \pm 5, \pm 2, \pm 1}$$

b) Use the graph of f in the figure shown and synthetic division to find **all** the zeros of the function. **(10 pts)**



-1	3	-11	9	13	-10
		-3	14	-23	10
$\frac{2}{3}$	3	-14	23	-10	$\boxed{0}$
		2	-8	10	
	3	-12	-15	$\boxed{0}$	

$$3x^2-12x+15=0$$
$$x^2-4x+5=0$$
$$x=\frac{4\pm\sqrt{16-4(1)(5)}}{2}=\frac{4\pm\sqrt{-4}}{2}=\frac{4\pm 2i}{2}=2\pm i$$

$$\boxed{\text{ZEROS: } -1, \tfrac{2}{3}, 2\pm i}$$

4. Find the equations of the asymptotes of $R(x)=\dfrac{2x^2+4x-5}{x^2-9}$ (do not graph $R(x)$) **(4 pts)**

$x^2-9=0$
$x^2=9$
$x=\pm 3$ $\longrightarrow$

horizontal asymptote $\underline{\quad y=2 \quad}$

vertical asymptote(s) $\underline{\quad x=-3 \text{ OR } 3 \quad}$

oblique asymptote $\underline{\quad \text{NONE} \quad}$

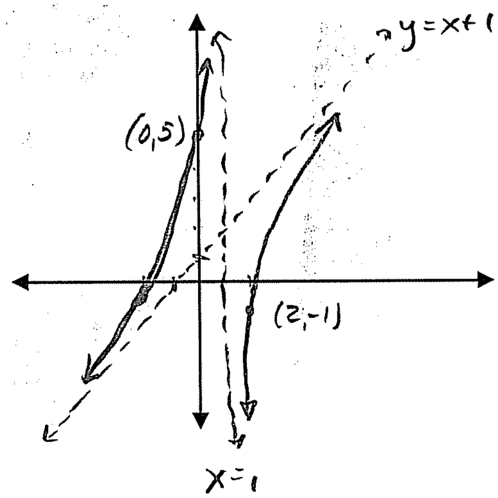
5. Sketch a graph of $R(x)=\dfrac{x^2-5}{x-1}$, label all asymptotes and at least two points on $R(x)$. **(8 pts)**

$$\begin{array}{r} x+1 \\ x-1\overline{)\,x^2+0x-5} \\ \underline{-x^2+x} \\ 1 \end{array}$$

SLANT $y=x+1$

$x=1$
VERTICAL
ASYMPTOTE

x	$R(x)$
0	5
-2	$-1/3$
2	-1



6. Evaluate the following determinant **(3 pts)**

$$\begin{vmatrix} -5 & -1 \\ -2 & -7 \end{vmatrix} = 35-2 = \boxed{33}$$

7. Use Cramer's rule to solve the system of equations **(5 pts)**

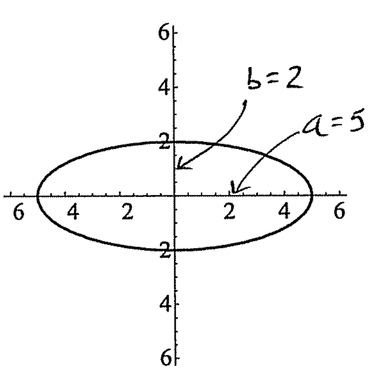
$3x+2y=2$
$2x+2y=3$

$$x=\frac{\begin{vmatrix} 2 & 2 \\ 3 & 2 \end{vmatrix}}{\begin{vmatrix} 3 & 2 \\ 2 & 2 \end{vmatrix}}=\frac{4-6}{6-4}=\frac{-2}{2}=-1$$

$$y=\frac{\begin{vmatrix} 3 & 2 \\ 2 & 3 \end{vmatrix}}{\begin{vmatrix} 3 & 2 \\ 2 & 2 \end{vmatrix}}=\frac{9-4}{6-4}=\frac{5}{2}$$

$\boxed{\text{SOLUTION } (-1, 5/2)}$

8. Write the standard form for the equation of the following ellipse and give the coordinates of its foci. **(5 pts)**



$$\boxed{\frac{x^2}{25}+\frac{y^2}{4}=1}$$

Foci: $c^2=25-4$
$c=\pm\sqrt{21}$ $\therefore$ $\boxed{(\pm\sqrt{21}, 0)}$

9. Sketch a graph of the following equation $4(y-3)^2-(x+2)^2=36$. Label vertices and show asymptotes and give the coordinates of the foci. **(8 pts)**

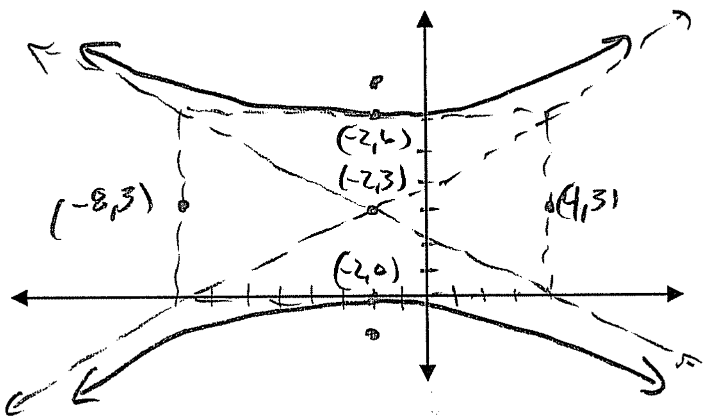
$\frac{(y-3)^2}{9}-\frac{(x+2)^2}{36}=1$ OPENS UP AND DOWN

CENTER $(-2,3)$

$c^2=9+36=45$

$c=\pm\sqrt{45}$ FOCI

$\boxed{(-2,\ 3\pm 3\sqrt{5}}$



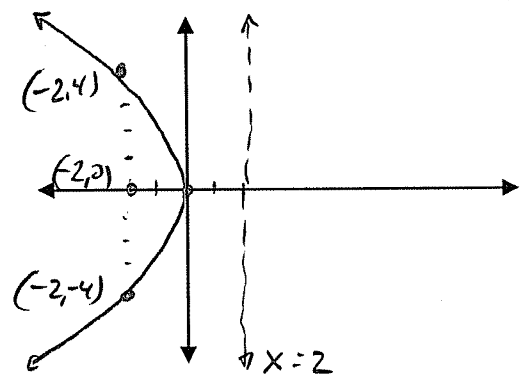
10. Sketch a graph of the following parabola $y^2=-8x$. Label the **focus** and **directrix** on the sketch along with **two other points** on the parabola. **(5 pts)**

VERTEX $(0,0)$

$4p=-8$

$p=-2$ FOCUS $(-2,0)$

DIRECTRIX $x=2$



11. Convert the equation to standard form by completing the square. Then, sketch a graph of the parabola labeling the focus and directrix on the sketch. **(8 pts)**

$x^2-2x-4y+9=0$

$x^2-2x+\boxed{1}=4y-9+\boxed{1}$

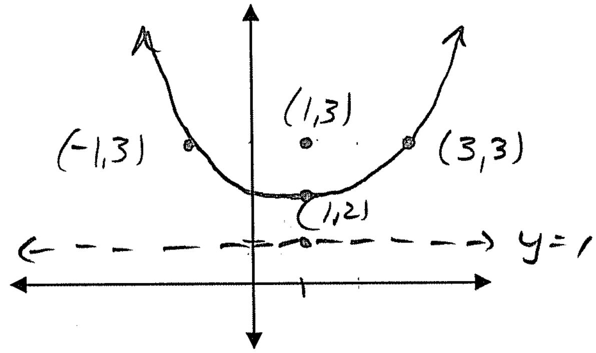
$(x-1)^2=4(y-2)$

VERTEX $(1,2)$

$4p=4$ FOCUS $(1,3)$

$p=1$

DIRECTRIX $y=1$



12. Write the following equation in standard form. Identify the equation as an ellipse, hyperbola or parabola. Finally, find the coordinates of the foci or focus. (No need to graph) **(6 pts)**

$9x^2 + 16y^2 - 18x + 64y - 71 = 0$

$9(x^2 - 2x + \boxed{1}) + 16(y^2 + 4y + \boxed{4}) = 71 + 9 + 64$

$\frac{9(x-1)^2}{144} + \frac{16(y+2)^2}{144} = \frac{144}{144}$

$\boxed{\frac{(x-1)^2}{16} + \frac{(y+2)^2}{9} = 1}$

ELLIPSE

CENTER $(1,-2)$

FOCI: $c^2 = 16 - 9$

$c = \pm\sqrt{7}$

COORDINATES

$\boxed{(1 \pm \sqrt{7}, -2)}$

13. Find the general solution to the following system **(5 pts)**

$$\begin{cases} x - 2y + z = 2 \\ 2x - y - z = 1 \end{cases}$$

$$\begin{bmatrix} 1 & -2 & 1 & 2 \\ 2 & -1 & -1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 1 & 2 \\ 0 & 3 & -3 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 1 & 2 \\ 0 & 1 & -1 & -1 \end{bmatrix} \to y - z = -1$$

LET $z = t$, $y = t - 1$

$x - 2(t-1) + t = 2$

$x - 2t + 2 + t = 2$ $\quad x = t$

$\boxed{\text{SOLUTION } (t, t-1, t)}$

5 POINTS EXTRA CREDIT: Make up a test question that you think should have been included on this test. **Solve** it and **explain** why this question should have been included. You cannot use an alteration of any of the existing test questions unless you have a legitimate explanation.

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Exam 1, Sec 3.3-3.5, 9.1, 9.2, 9.5, 10.1-10.3 Blitzer, 4th ed.
Spring 2010 Bordwell **20 pts**

This is the take home portion of Exam 1. You cannot use Tutors or Higher Ed Tutors for help. You may work on this with other students from the class, but you need to individually hand in your own tests. Include all steps to solving each problem. Hand in at the beginning of class on Thursday, Feb 4, 2010.

1. Solve the following system of equations using matrices. Use Gaussian elimination with back-substitution or Gauss-Jordan elimination. Show every step to receive full credit. **(5 pts)**

$$2x+y=2z+4$$
$$2x=1+y-4z$$
$$x+y+6z=7$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 6 & 7 \\ 2 & 1 & -2 & 4 \\ 2 & -1 & 4 & 1 \end{array}\right] \rightarrow \begin{bmatrix} 1 & 1 & 6 & 7 \\ 0 & -1 & -14 & -10 \\ 0 & -3 & -8 & -13 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 6 & 7 \\ 0 & 1 & 14 & 10 \\ 0 & 0 & 34 & 17 \end{bmatrix}$$

$34z = 17$, $\boxed{z = \tfrac{1}{2}}$

$y + 14(\tfrac{1}{2}) = 10$, $\boxed{y = 3}$

$x + 3 + 6(\tfrac{1}{2}) = 7$

$x + 6 = 7$

$\boxed{x = 1}$

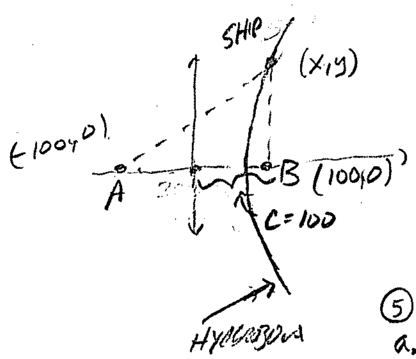
$\boxed{\text{SOLUTION } (1, 3, \tfrac{1}{2})}$

2. a) Use your graphing calculator to find the determinant. $\boxed{-2100}$ **(4 pts)**

$$\begin{vmatrix} 3 & -2 & -1 & 4 \\ -5 & 1 & 2 & 7 \\ 2 & 4 & 5 & 0 \\ -1 & 3 & -6 & 5 \end{vmatrix}$$

b) Use your calculator to write the matrix in a) above in **row-echelon** form. Transform the components in the matrix you find to fractions. Write your answers for *x*, *y* and *z*. $\boxed{\text{NO SOLUTION}}$

3. Do problem #62 from **Sec 10.2**. Show all work and any sketches you make to receive full credit. **(6 pts)**



SINCE B RECEIVED THE SIGNAL 500 MICROSECONDS BEFORE A THEN THE DIFFERENCE IN DISTANCES IS 300(500) = 150,000m or 150km ①

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ SINCE $2a = 150$ ②, $a = 75$

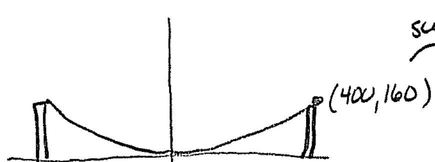
③ $\frac{x^2}{75^2} - \frac{y^2}{b^2} = 1$, ④ $100^2 = 75^2 + b^2$, $b^2 = 100^2 - 75^2$ or $b = 4375$

⑤ EQUATION: a.) $\boxed{\frac{x^2}{5625} - \frac{y^2}{4375} = 1}$

⑥ LET $x = 100$, FIND y: $\frac{100^2}{5625} - \frac{y^2}{4375} = 1$, $y^2 = \left(\frac{100^2}{5625} + 1\right)4375$

b.) $\boxed{y \approx 58.3 \text{ MILES}}$

4. Do Problem #66 from Sec **10.3**. Show all work and any sketches you make to receive full credit. **(5 pts)**



$x^2 = 4py$

sub. $\rightarrow 400^2 = 4p(160)$

$\frac{400^2}{4(160)} = p = 250$

$\therefore \boxed{x^2 = 1000y}$ EQUATION

100 ft FROM THE TOWER WOULD BE $x = 300$

$300^2 = 1000y$

$\frac{90000}{1000} = \boxed{y = 90 \text{ ft}}$