

Work neatly and show all algebraic and numerical work.

1. Convert the angle 300° to radians. Express your answer as a multiple of π . (3pts)

$$300^\circ \times \frac{\pi}{180} = \boxed{\frac{5\pi}{3}}$$

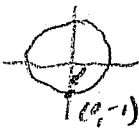
2. Convert the angle $-\frac{\pi}{9}$ to a positive angle in degrees. (3 pts)

$$-\frac{\pi}{9} \times \frac{180}{\pi} = -20^\circ \text{ or } \boxed{340^\circ}$$

3. Find the exact value of each of the following trig functions: (21 pts)

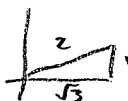
a.) $\cos(-90^\circ) = \boxed{0}$

(2)



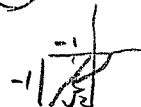
b.) $\sin(\pi/6) = \boxed{\frac{1}{2}}$

(2)



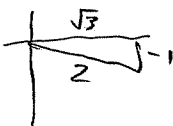
c.) $\tan(-3\pi/4) = \boxed{1}$

(2)



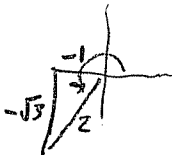
d.) $\cos(11\pi/6) = \boxed{\frac{\sqrt{3}}{2}}$

(2)



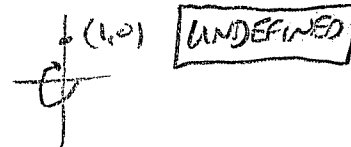
e.) $\sin(240^\circ) = \boxed{-\frac{\sqrt{3}}{2}}$

(2)



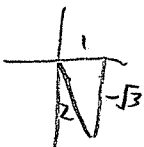
f.) $\tan(-270^\circ) = \frac{1}{0}$

(2)



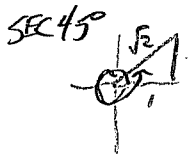
g.) $\sin(5\pi/3) = \boxed{-\frac{\sqrt{3}}{2}}$

(2)



h.) $\sec(405^\circ) = \boxed{\sqrt{2}}$

(3)

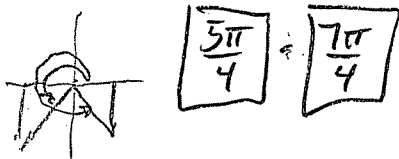


i.) $\sin^2(5\pi/6) - \tan^2(-\pi/6)$

(4)

$$\left(\frac{1}{2}\right)^2 - \left(-\frac{1}{\sqrt{3}}\right)^2 = \frac{1}{4} - \frac{1}{3} = \frac{3-4}{12} = \boxed{-\frac{1}{12}}$$

4. Find two values of θ , $0 \leq \theta \leq 2\pi$, that satisfies the equation $\sin \theta = -\frac{\sqrt{2}}{2}$. (4 pts)



5. Use trigonometric identities to find the exact value of each of the following expressions: (4 pts)

a.) $\cos(53^\circ)\sec(53^\circ)$

$\cos(53^\circ) \cdot \frac{1}{\cos(53^\circ)} = \boxed{1}$

b.) $\sin^2\left(\frac{\pi}{10}\right) + \cos^2\left(\frac{\pi}{10}\right)$

$\boxed{1}$

6. Let $\theta = -4\pi/3$. Sketch the angle on the unit circle provided. Draw the reference triangle and indicate the reference angle on the sketch. Find the values of the six trigonometric functions for this angle. (12 pts)

$\sin(\theta) = \boxed{\frac{\sqrt{3}}{2}}$

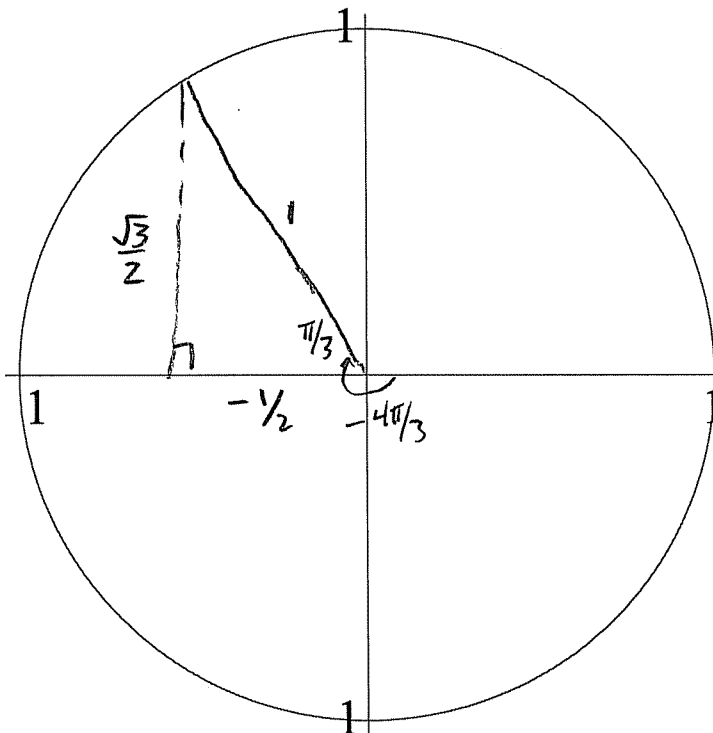
$\cos(\theta) = \boxed{-\frac{1}{2}}$

$\tan(\theta) = \boxed{-\sqrt{3}}$

$\csc(\theta) = \boxed{\frac{2}{\sqrt{3}}}$

$\sec(\theta) = \boxed{-2}$

$\cot(\theta) = \boxed{-\frac{1}{\sqrt{3}}}$



7. Find the **amplitude, period, phase shift** and **vertical shift** of the following functions. (16 pts)

a.) $y = 15 \cos(3t + \pi/6) - 5$

$$0 \leq 3t + \frac{\pi}{6} \leq 2\pi$$

$$-\frac{\pi}{6} \div 3 \quad -\frac{\pi}{18}$$

amplitude = 15

period = $\frac{2\pi}{3}$

phase shift = $-\frac{\pi}{18}$

vertical shift = -5 or Down 5

b.) $y = -2 \sin\left(t - \frac{\pi}{3}\right)$

amplitude = 2

period = 2π

phase shift = $\frac{\pi}{3}$

vertical shift = 0

8. Sketch one period of the graph of the equation in $y = 2 \cos(4t + \pi) - 1$ on the axis provided.

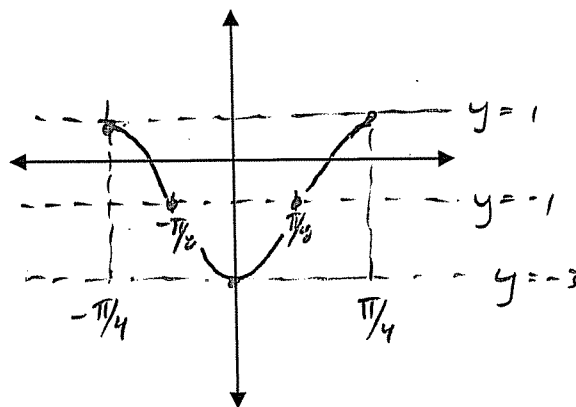
Draw a characteristic "box" and label all tick marks on the x- and y-axis. (6 pts)

Amp = 2

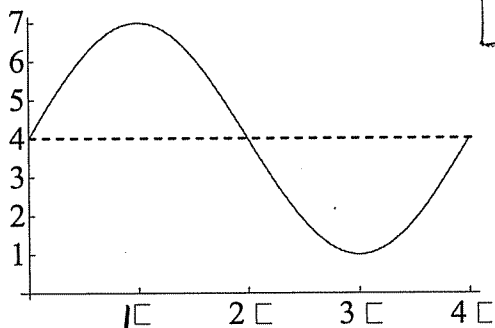
$$0 \leq 4t + \pi \leq 2\pi$$

$$-\pi \leq 4t \leq \pi$$

$$-\frac{\pi}{4} \leq t \leq \frac{\pi}{4}$$



9. Find the equation in the form $y = A \sin(Bx) + D$ and $y = A \cos(Bx - C) + D$ that produces the graph below. (6 pts)



$$y = 3 \sin\left(\frac{\pi}{2}x\right) + 4$$

$$\frac{2\pi}{B} = 4$$

$$B = \frac{\pi}{2}$$

$$y = 3 \cos\left(\frac{\pi}{2}x - \frac{\pi}{2}\right) + 4$$

OR

$$y = -3 \cos\left(\frac{\pi}{2}x + \frac{\pi}{2}\right) + 4$$

**Turn in this portion of the test to receive the Calculator portion of the exam.

Show all problem solving steps to receive full credit. Round all answers to 4 decimal places when appropriate.

1. Convert $30^{\circ}15'10''$ to a decimal in degrees.

$$\boxed{30.2528^{\circ}}$$

(2pts)

2. Convert 50.4256° to degrees-minutes-seconds (DMS).

$$\boxed{50^{\circ} 25' 32.16''}$$

(2 pts)

3. Find the arc length on a circle of radius 20 feet cut off by a central angle of 39° . Hint: use the formula $s = r\theta$.

(3pts)

$$39^{\circ} \times \frac{\pi}{180^{\circ}} \quad s = 20 \left(\frac{39\pi}{180} \right) = \boxed{\frac{39\pi}{9} \text{ feet} \approx 13.6136}$$

4. A water wheel has a radius of 12 feet. The wheel is rotating at 20 revolutions per minute. Use the following formulas

$$v = r\omega \text{ where } v = \frac{s}{t} \text{ and } \omega = \frac{\theta}{t} \text{ to answer the following questions.}$$

- a) Find the angular speed in radians per minute.

(3 pts)

$$\omega = \frac{20 \text{ REV}}{\text{MIN}} \times \frac{2\pi}{1 \text{ REV}} = \boxed{\frac{40\pi \text{ RAD}}{\text{MIN}} \approx 125.6637}$$

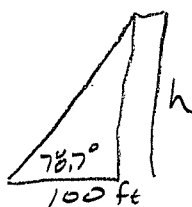
- b) Find the linear speed, in feet per minute, of the water.

(3 pts)

$$v = 12 \times 40\pi = \boxed{480\pi \text{ ft/min} \approx 1507.9645 \text{ ft/min}}$$

5. The angle of inclination to the top of a building is 78.7° from a point 100 feet from the base of the building and on level ground. Find the height of the building to the nearest foot.

(4 pts)



$$\tan 78.7^{\circ} = \frac{h}{100}$$

$$\boxed{100 \tan 78.7^{\circ} = h \approx 500 \text{ ft}}$$

NOTE: 16.16 IF RADIAN MODE

6. Find the value of the acute angle θ where $\cos \theta = 0.4122$. State your answer in

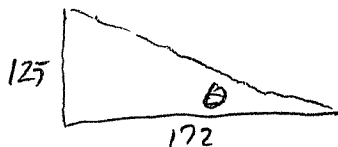
(4 pts)

a) degrees $\boxed{65.6569^{\circ}}$

b) radians $\boxed{1.1459}$

7. A tower that is 125 feet tall casts a shadow 172 feet long. Find the angle of elevation of the sun to the nearest degree.

(4 pts)



$$\tan \theta = \frac{125}{172}$$

$$\boxed{\theta = \tan^{-1} \left(\frac{125}{172} \right) \approx 36^{\circ}}$$

NOTE: 0.62845 IN RADIAN MODE

5 POINTS EXTRA CREDIT: On the back of this test, make up a test question that you think should have been included on this test. Solve it and explain why this question should have been included. You cannot use an alteration of any of the existing test questions unless you have a legitimate explanation.