MATH 2210 – Differential Equations Spring 2010
Problem Set #2 – Due by 3:00 p.m. on Tuesday, February 16, 2010

Suppose that $\frac{dP}{dt} = 0.3P\left(1 - \frac{P}{2500}\right)$ is a model for a fish population. Refer to exercise 15 in section 1.1 in helping you to answer the following questions.

1) Let's see what happens if a certain number of fish are harvested each year.

a) Draw phase lines for harvest rates of 120, 180, and 187.2 (whatever that means) fish per year.

b) How would you describe the effect that different harvest rates have on the long-term fish population for various initial populations?

c) Find the harvest rate that results in just one equilibrium solution and draw the corresponding phase line. What happens to the fish population in the long-term in this scenario for various initial populations?

d) What happens to the fish population if the harvest rate is increased even further? Why?

e) Solve the initial value problem analytically (with technology if you want) with a harvest rate of 120 fish per year and an initial population of 600 fish. Then use your answer to estimate the fish population in 30 years.

f) Use Euler's Method (with technology) to get a good estimate of the fish population in 30 years if 600 fish are present initially and 120 fish are harvested per year. (Do NOT print out pages and pages of data.) To the nearest hundred, what is the smallest number of steps that gives you an answer within 0.1 of your answer from part e? What is the corresponding step size?

g) Use the Runge-Kutta Method (with technology) to get a good estimate of the fish population in 30 years if 600 fish are present initially and 120 fish are harvested per year. (Do NOT print out pages and pages of data.) To the nearest integer, what is the largest step size that gives you an answer within 0.1 of your answer from part e? What is the corresponding number of steps?

2) Now let's see what happens if different fractions of the population are harvested each year.

a) Draw phase lines for harvest rates of 1/4, 1/5, and 1/6 of the population per year.

b) How would you describe the effect that harvesting different fractions of the population per year has on the long-term fish population for various initial populations? How does this compare with what happened in part 1b?

c) What fraction of the population would have to be harvested per year so that there would be only 1 equilibrium solution? What happens to the fish in the long-term in this scenario for various initial populations?

d) Solve the initial value problem analytically (with technology if you want) with 1/4 of the fish being harvested per year and an initial population of 600 fish. Then use your answer to estimate the fish population in 30 years.

e) Use Euler's Method (with technology) to get a good estimate of the fish population in 30 years if 600 fish are present initially and 1/4 of the fish are harvested per year. (Do NOT print out pages and pages of data.)

f) Use the Runge-Kutta Method (with technology) to get a good estimate of the fish population in 30 years if 600 fish are present initially and 1/4 of the fish are harvested per year. (Do NOT print out pages and pages of data.)