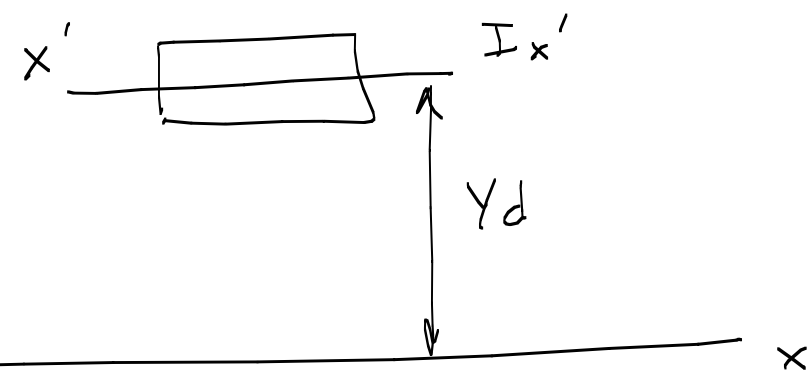
$$I_y = \int \tilde{x}^2 \, dA \qquad I_x = \int \tilde{y}^2 \, dA$$

Parallel-Axis Theorem

$$I_x = I_{x'} + A y_d^2$$

$$I_y = I_{y'} + A x_d^2$$



distance between the x' and x axes

For multiple shapes

$$I_x = \Sigma\left(I_{x'} + A y_d^2\right)$$

$$I_y = \Sigma\left(I_{y'} + A x_d^2\right)$$

Used for composite shapes

Moment of Inertias of simple shapes, and transform using the parallel-axis theorem

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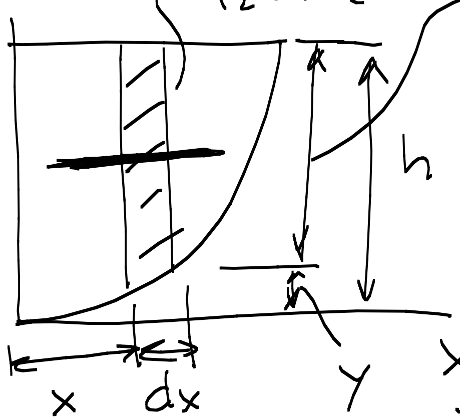
Note on differential strips

$I = \frac{1}{12} dx\,(h-y)^3$ $h-y$ $\cancel{I_x = \int \tilde{y}^2 dA}$

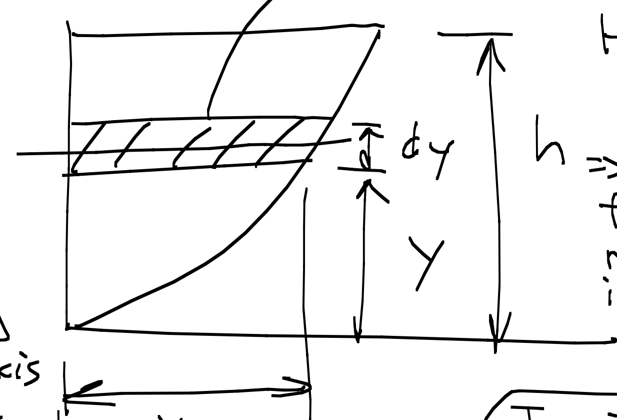
Find I_x



Find I_x for the element, and transform using the parallel-axis theorem, and integrate

$\frac{1}{12} x (dy)^3$

Higher Order Term



$\Rightarrow$ Don't need to transform the moment of inertia of the strip

$$\boxed{I_x = \int \tilde{y}^2 dA}$$

$$I_{rectangle} = \frac{1}{12} b h^3$$

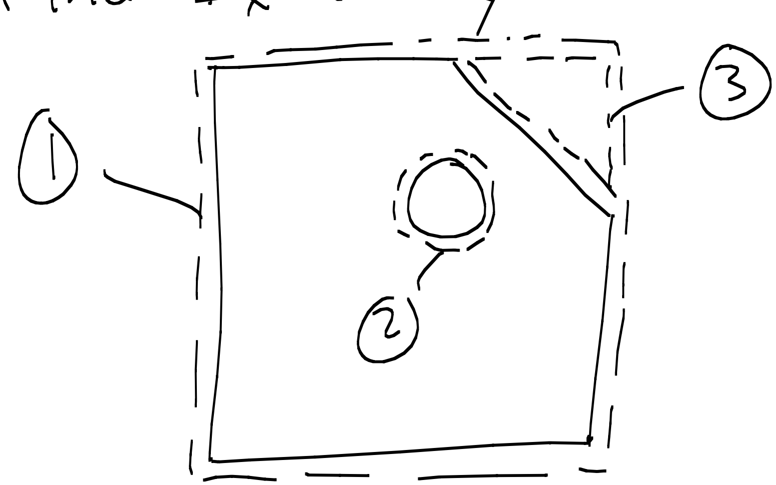
parallel axis theorem adds $A y_d^2$

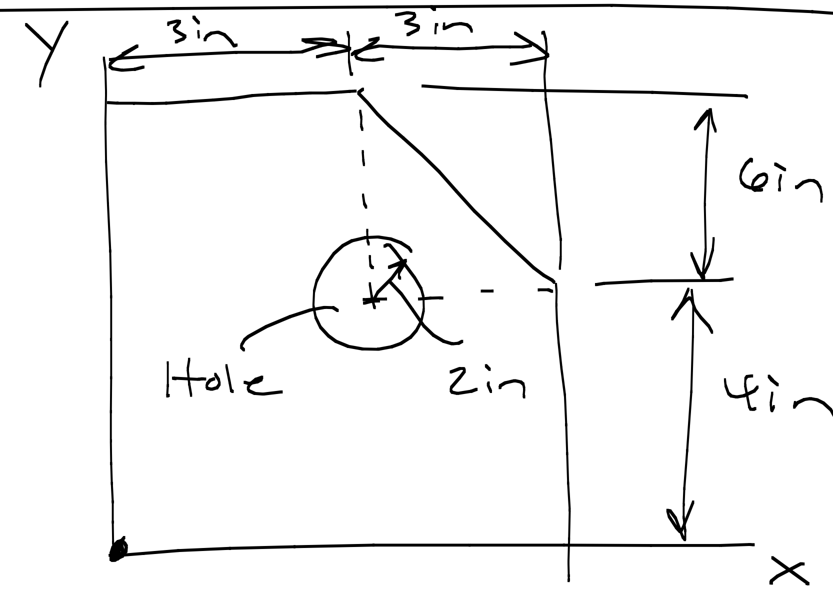
Select a strip that is parallel to axis about which you want to find the moment of inertia.

Otherwise, you must find I for the differential element, and transform using the parallel-axis theorem

Example

Find I_x & I_y





Shape	$I_{x'}$ (in^4)	$I_{y'}$ (in^4)	A (in^2)	x_d (in)	y_d (in)	$x_d^2 A$ (in^4)	$y_d^2 A$ (in^4)
1	$\frac{1}{12}(6)(10)^3 = 500$	$\frac{1}{12}(10)(6)^3 = 180$	$(6)(10) = 60$	3 in	5 in	540	1500
2	$-\frac{1}{4}\pi(2)^4$ -12.57	-12.57	$-\pi(2)^2 = -12.57$	3 in	4 in	-113.13	-201.12
3	$-\frac{1}{36}(3)(6)^3$ -18	$-\frac{1}{36}(6)(3)^3$ -4.5	$-\frac{1}{2}(3)(6) = -9$	$3+\frac{2}{3}(3) = 5$	$4+\frac{2}{3}(6) = 8$	-225	-576
	$\Sigma I_{x'} = 469.43$	$\Sigma I_{y'} = 162.93$				$\Sigma x_d^2 A = 201.87$	$\Sigma y_d^2 A = 722.88$

$$I_x = \Sigma(I_{x'} + y_d^2 A) = 469.43\,in^4 + 722.88\,in^4 = \boxed{1{,}192.31\,in^4}$$

$$I_y = \Sigma(I_{y'} + x_d^2 A) = 162.93\,in^4 + 201.87\,in^4 = \boxed{364.8\,in^4}$$