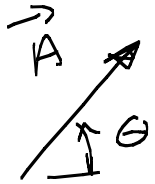
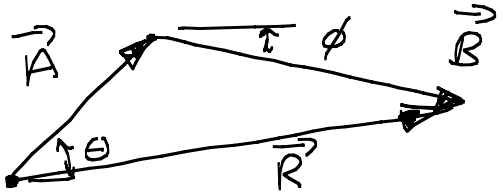
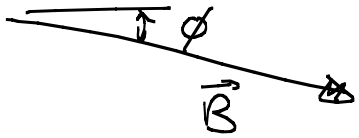
Triangle Construction

Connect Head to Tail

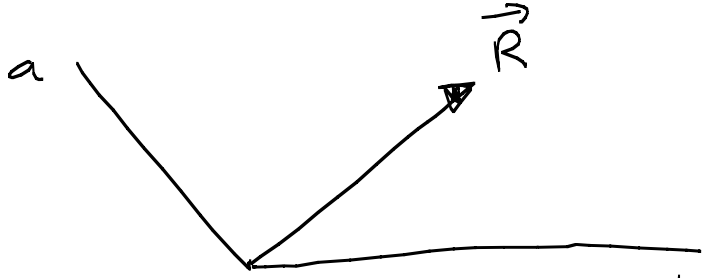






Resolving Vectors into Components

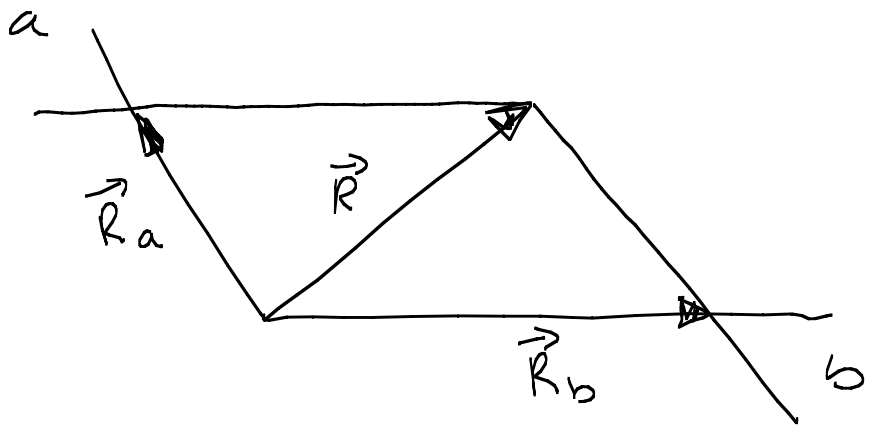
Given: - The resultant vector $\vec{R}$
- Lines of Action a & b



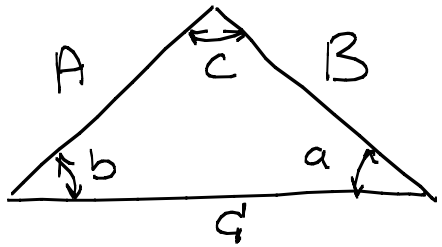
Find: Components along a & b

Procedure

- Draw extension lines from the head of $\vec{R}$ that are parallel to a and to b
- Where the extension lines cross the a & b axes represents the head of each component vector



Law of Sines and Cosines



Law of Sines

$$\frac{\sin a}{A} = \frac{\sin b}{B} = \frac{\sin C}{C}$$

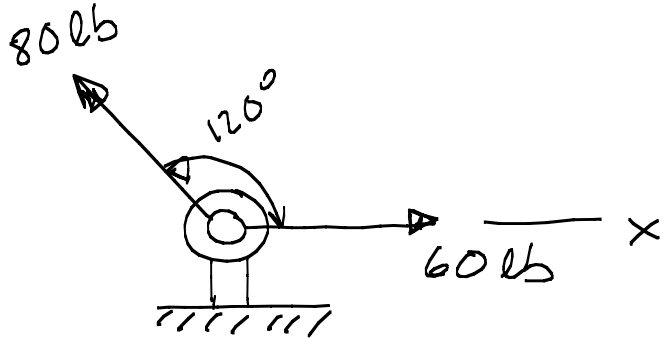
Law of Cosines

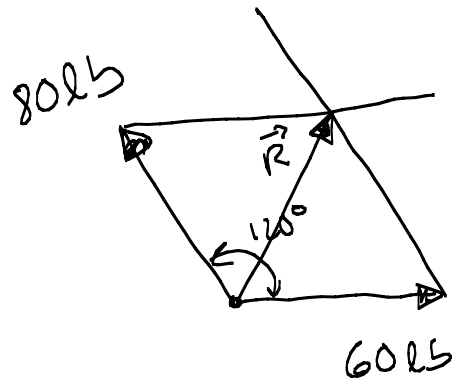
$$C^2 = A^2 + B^2 - 2AB\cos(c)$$

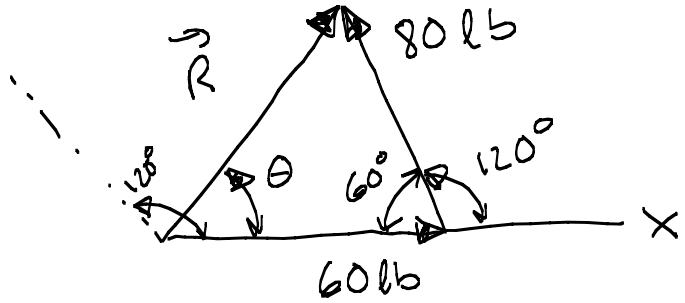
Example

Determine

The magnitude of the resultant force, and its direction measured counter-clockwise from the positive x-axis







$$R^2 = (80lb)^2 + (60lb)^2 - 2(80lb)(60lb)\cos(60^\circ)$$

$$\boxed{R = 72.1\ lb}$$

Law of Sines

$$\frac{80\ lb}{\sin\theta} = \frac{72.1\ lb}{\sin 60^\circ}$$

$$\boxed{\theta = 73.9^\circ}$$

$\vec{R} = 72.1\ lb$ 73.9°

Rectangular Components — 90° Axes

Advantage ~~Law of Sines~~ ~~Law of Cosines~~

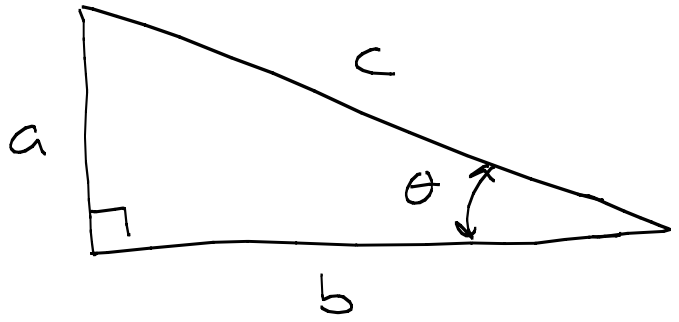
Add more than two forces at a time

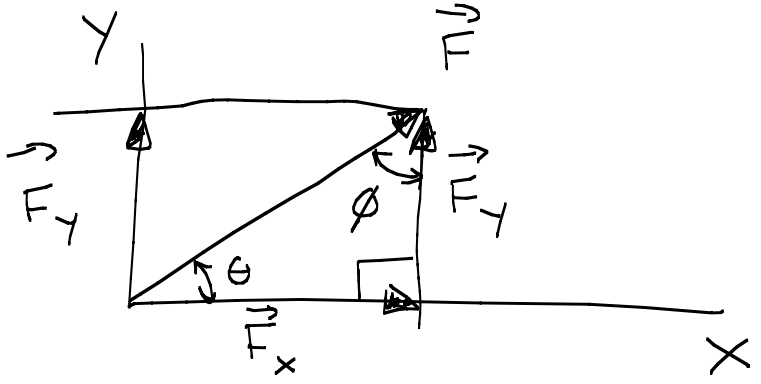
Pythagorean Theorem

$$c^2 = a^2 + b^2$$

$$a = c \sin\theta$$

$$b = c(\cos\theta)$$





$$F_x = F\cos\theta$$

$$F_y = F\sin\theta$$

$$F_x = F\sin\phi$$

$$F_y = F\cos\phi$$

Cartesian Vector Notation

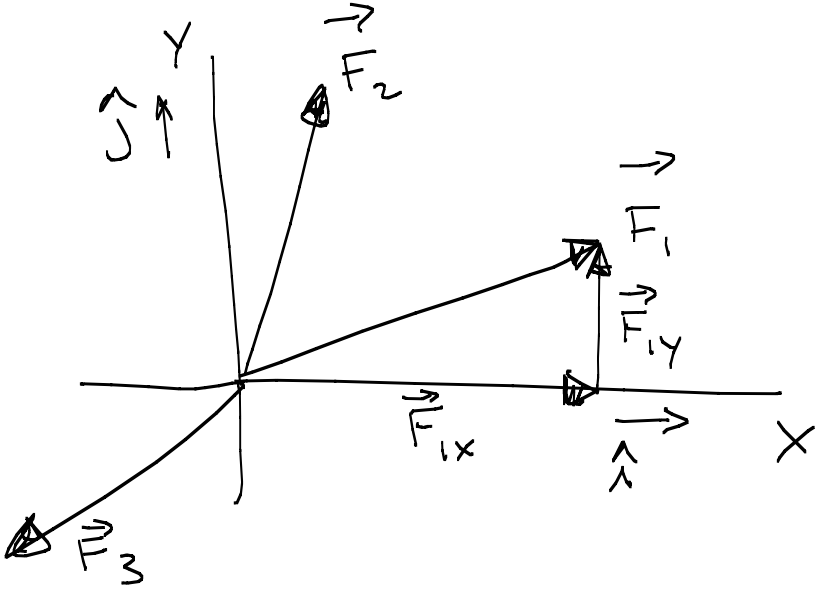
Cartesian Unit Vectors $\hat{i}$ & $\hat{j}$

$\vec{F}_x = F_x \hat{i}$ — F_x: scalar; $\hat{i}$: unit vector

$\vec{F}_y = F_y \hat{j}$

$$\boxed{\vec{F} = F_x \hat{i} + F_y \hat{j}}$$

Resultant Force, $\vec{F}_R$



$$\vec{F}_R = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$$

$$\vec{F}_1 = F_{1x} \hat{i} + F_{1y} \hat{j}$$

$$\vec{F}_2 = F_{2x} \hat{i} + F_{2y} \hat{j}$$

$$\vec{F}_3 = -F_{3x} \hat{i} - F_{3y} \hat{j}$$

Add the $\hat{i}$-comp & add the $\hat{j}$-comp

$$\vec{F}_R = \left(F_{1x} + F_{2x} - F_{3x}\right)\hat{i} + \left(F_{1y} + F_{2y} - F_{3y}\right)\hat{j}$$