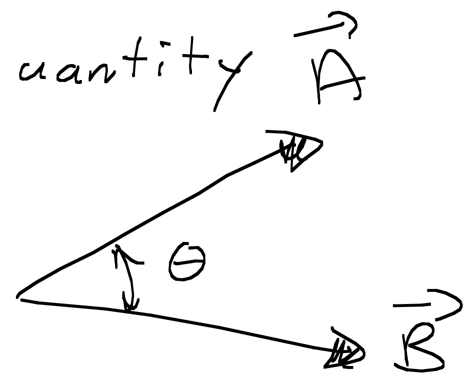
Dot Product

- Product of the magnitudes of two vectors
- Used to determine the angle between two vectors
- Dot Product is a scalar quantity



$$\vec{A} \bullet \vec{B} = AB\cos\theta$$

Dot Product

Laws of Operation

1) $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$

2) $\underset{\text{scalar}}{a}(\vec{A} \cdot \vec{B}) = (a\vec{A}) \cdot \vec{B} = \vec{A} \cdot (a\vec{B}) = (\vec{A} \cdot \vec{B})\, a$

3) $\vec{A} \cdot (\vec{B} + \vec{D}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{D}$

For Cartesian Vectors

$$|\hat{i}| = |\hat{j}| = |\hat{k}| = 1$$

$$\vec{A} \cdot \vec{B} = AB\cos\theta$$

$$\hat{i} \cdot \hat{i} = (|\hat{i}|)(|\hat{i}|)\cos(0) = (1)(1)(1) = 1$$

$\underset{\hat{i}}{\longrightarrow}$ $\underset{\hat{i}}{\longrightarrow}$

$$\hat{j} \cdot \hat{j} = (1)(1)(\cos(0)) = 1 \qquad \hat{k} \cdot \hat{k} = 1$$

$$\hat{i} \cdot \hat{j} = (1)(1)\cos(90^\circ) = 0$$

$$\hat{i} \cdot \hat{k} = 0, \quad \hat{k} \cdot \hat{j} = 0$$

For Cartesian vectors, the only non-zero terms occur when the Cartesian vectors are dot producted with themselves

$$\vec{A} \cdot \vec{B} = (A_x \hat{i} + A_y \hat{j} + A_z \hat{k}) \cdot (B_x \hat{i} + B_y \hat{j} + B_z \hat{k})$$

$$= A_x B_x (\hat{i} \cdot \hat{i}) + A_x B_y \cancelto{0}{(\hat{i} \cdot \hat{j})} + A_x B_z \cancelto{0}{(\hat{i} \cdot \hat{k})}$$

$$+ A_y B_x \cancelto{0}{(\hat{j} \cdot \hat{i})} + A_y B_y (\hat{j} \cdot \hat{j}) + A_y B_z \cancelto{0}{(\hat{j} \cdot \hat{k})}$$

$$+ A_z B_x \cancelto{0}{(\hat{k} \cdot \hat{i})} + A_z B_y \cancelto{0}{(\hat{k} \cdot \hat{j})} + A_z B_z (\hat{k} \cdot \hat{k})$$

$$\boxed{\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z}$$

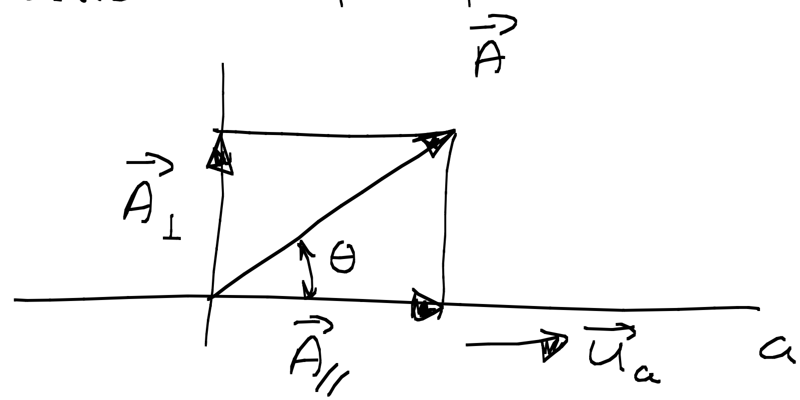
Uses for the Dot Product

1) Angle between two vectors

$$\vec{A} \cdot \vec{B} = AB\cos\theta$$

$$\theta = \cos^{-1}\left(\frac{\vec{A}\cdot\vec{B}}{AB}\right)$$

2) Vector components parallel and perpendicular to a line



$$\vec{A}_{\parallel} = \vec{A}\cos\theta$$

$$\vec{A}_{\parallel} = A\vec{u}_a\cos\theta$$

$$\vec{A}_{\parallel} = A\cos\theta\,\vec{u}_a$$

$$\vec{A} \cdot \vec{u}_a = A(1)\cos\theta = A\cos\theta$$

$$\boxed{\begin{aligned} \vec{A}_{\parallel} &= (\vec{A} \cdot \vec{u}_a)\,\vec{u}_a \\ A_{\parallel} &= (\vec{A} \cdot \vec{u}_a) \end{aligned}}$$

$$A_{\perp} = \sqrt{A^2 - A_{\parallel}^2}$$

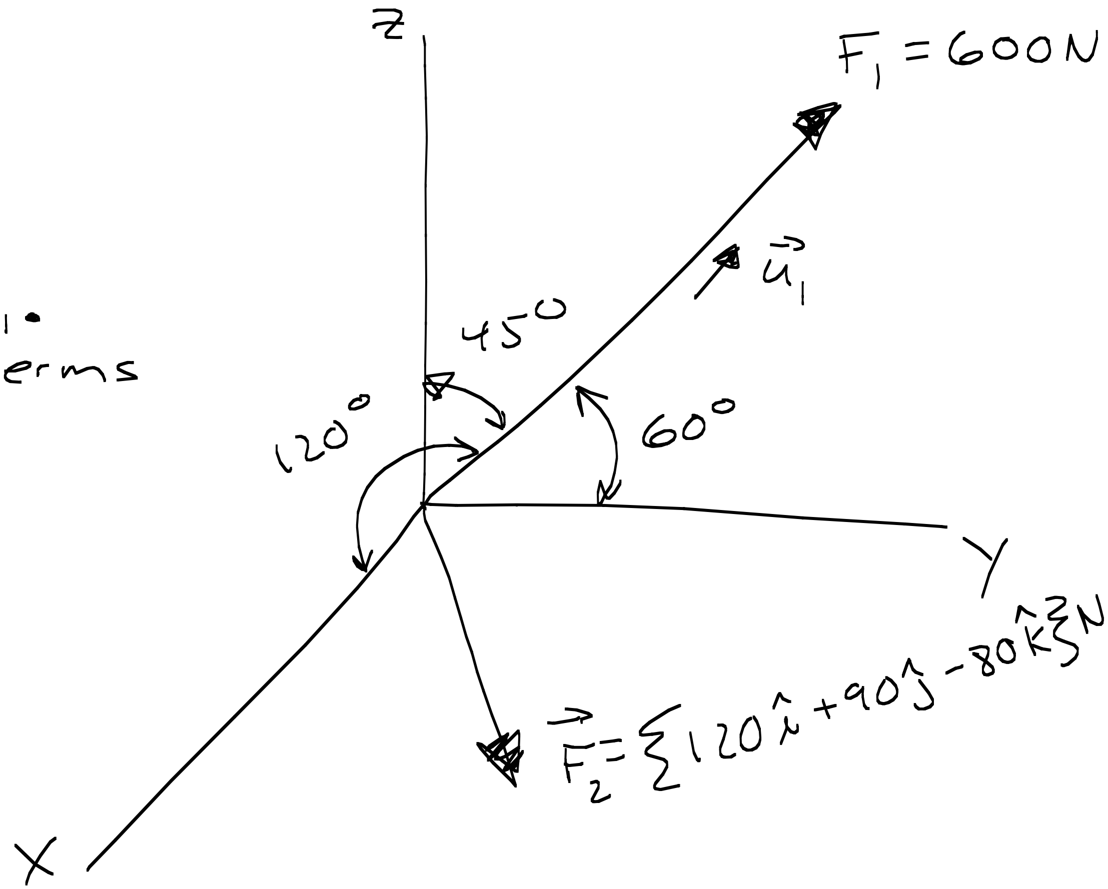
Steps to find components of a vector parallel to a line

1. Find the unit vector of the line
2. Take the Dot Product of the force vector and the unit vector from 1 => Scalar
3. Multiply the scalar by the unit vector from 1

Example

Determine

The component of $\vec{F}_2$ that is directed along $\vec{F}_1$. Express you answer in terms of a Cartesian vector



(1) $\vec{u}_1 = \{\cos\alpha\,\hat{i} + \cos\beta\,\hat{j} + \cos\gamma\,\hat{k}\}$

$= \{\cos 120^o\,\hat{i} + \cos 60^o\,\hat{j} + \cos 45\,\hat{k}\}$

$\vec{u}_1 = \{-0.5\,\hat{i} + 0.5\,\hat{j} + 0.707\,\hat{k}\}$

(2) $\vec{F}_2 \cdot \vec{u}_1 = \{120\hat{i} + 90\hat{j} - 80\hat{k}\}N \cdot \{-0.5\hat{i} + 0.5\hat{j} + 0.707\hat{k}\}$

$$= [(120)(-0.5) + (90)(0.5) + (-80)(0.707)]\,N$$

$$= -71.56\ N$$

In cartesian form

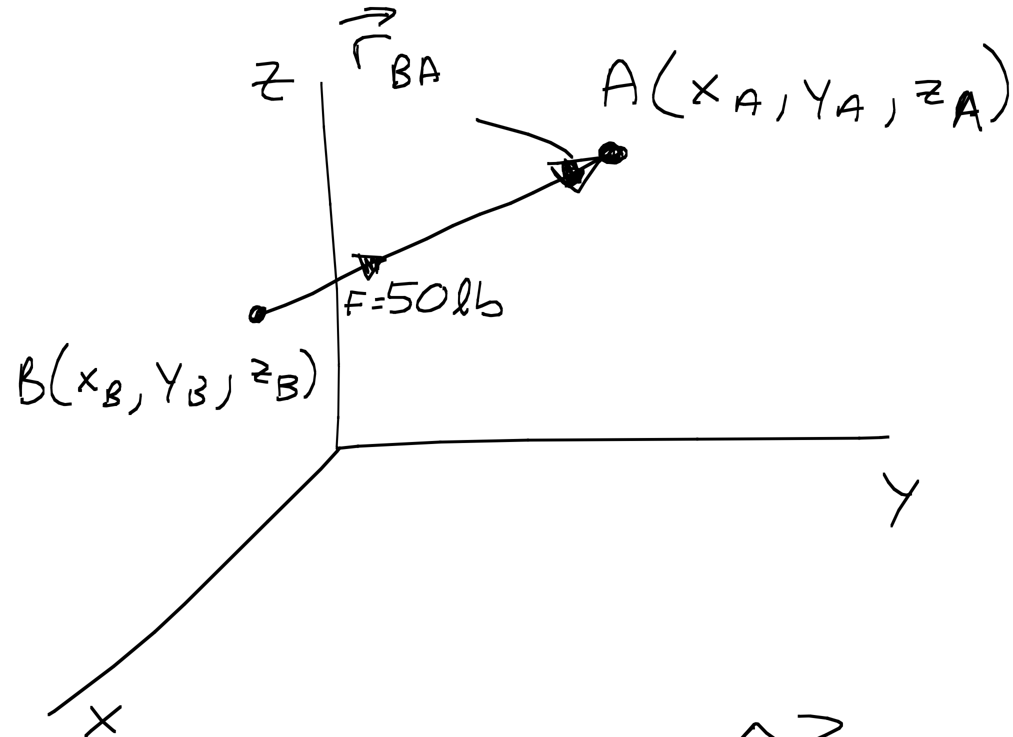
(3) $\Rightarrow (-71.56N)\,\vec{u}_1 = (-71.56N)\{-0.5\hat{i} + 0.5\hat{j} + 0.707\hat{k}\}$

$$= \boxed{\{35.78\hat{i} - 35.78\hat{j} - 50.59\hat{k}\}N}$$

Position Vectors

$\vec{r}_{BA}$ = relative position vector

Arrow => B to A



$$\vec{r}_{BA} = \{(x_A - x_B)\hat{i} + (y_A - y_B)\hat{j} + (z_A - z_B)\hat{k}\}$$

$$\vec{u}_{BA} = \frac{\vec{r}_{BA}}{r_{BA}} \qquad r_{BA} = \sqrt{(r_{BA_x})^2 + (r_{BA})_y^2 + (r_{BA_z})^2}$$

$$\vec{F} = (50\,lb)(\vec{u}_{BA})$$