

Chapter 3: Equilibrium of a Particle

Equilibrium

Newton's 2nd $\vec{F}_R = m\vec{a}$

Static Equilibrium $\vec{a} = 0$

$$\vec{F}_R = 0 \Rightarrow \boxed{\sum \vec{F} = 0}$$

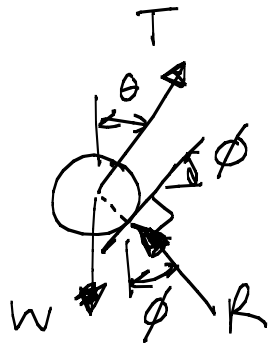
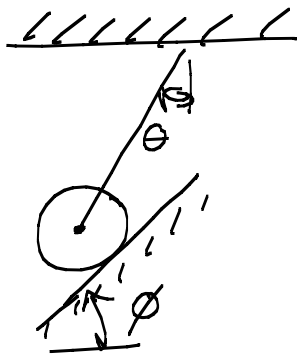
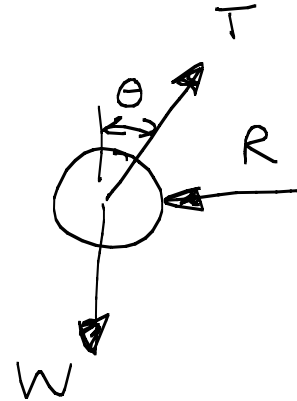
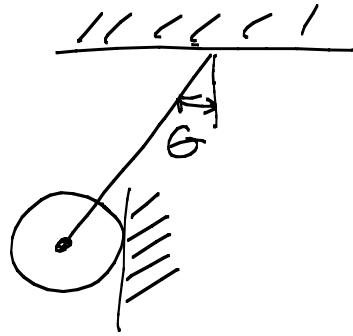
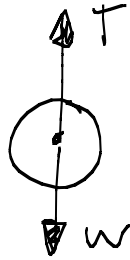
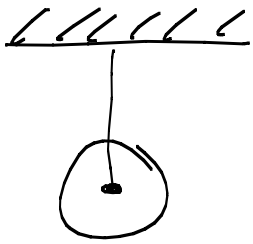
Free-Body Diagram (FBD)

\Rightarrow Shows a sketch of the body removed from its surroundings with all of the external forces acting on it

- Examples:
- Weight
 - Contact surface
 - Support system (cable, rope, etc.)

Additional Assumptions

- Contact Surfaces are smooth (Neglect Friction)
- Bending Resistance of support systems is negligible



Solving for Unknown Forces

$$\sum \vec{F} = 0$$

$$\sum F_x \hat{i} + \sum F_y \hat{j} = 0$$

Equilibrium Equations

$$\sum F_x = 0$$

$$\sum F_y = 0$$

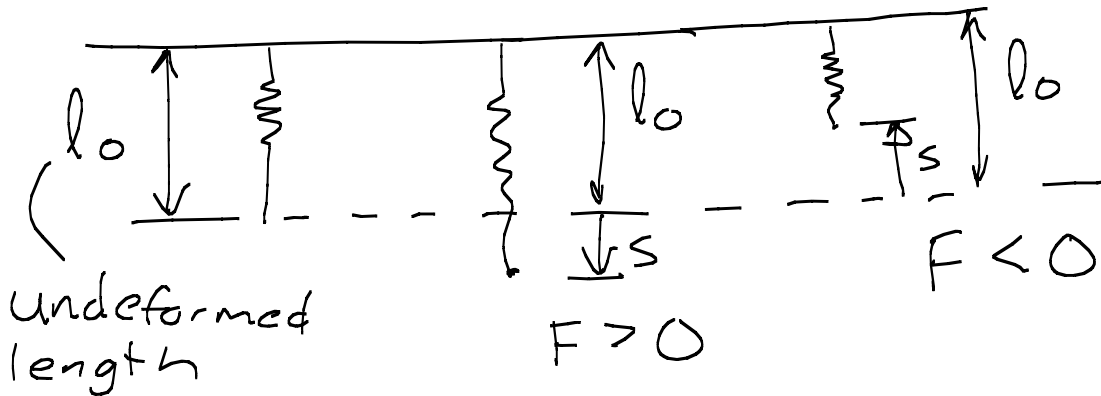
For Each FBD, we get two equilibrium equations \Rightarrow can solve for two unknowns
If the solution for an unknown force yields a negative value, then the sense of direction of the force is opposite of what was assumed in the FBD

Springs

$$F = k s$$

spring
stiffness

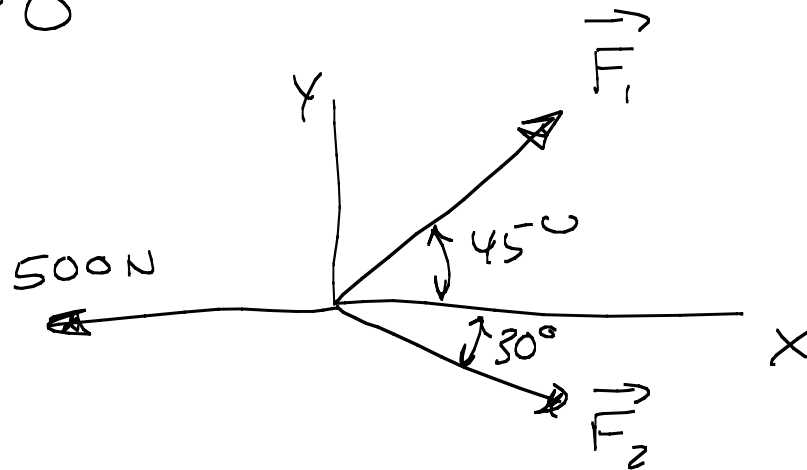
difference in the spring's deformed
length from its undeformed
length



Example

Determine:

$$F_1 + F_2$$



$$\rightarrow \sum F_x = 0 \Rightarrow F_1 \cos 45^\circ + F_2 \cos 30^\circ - 500 \text{ N} = 0$$

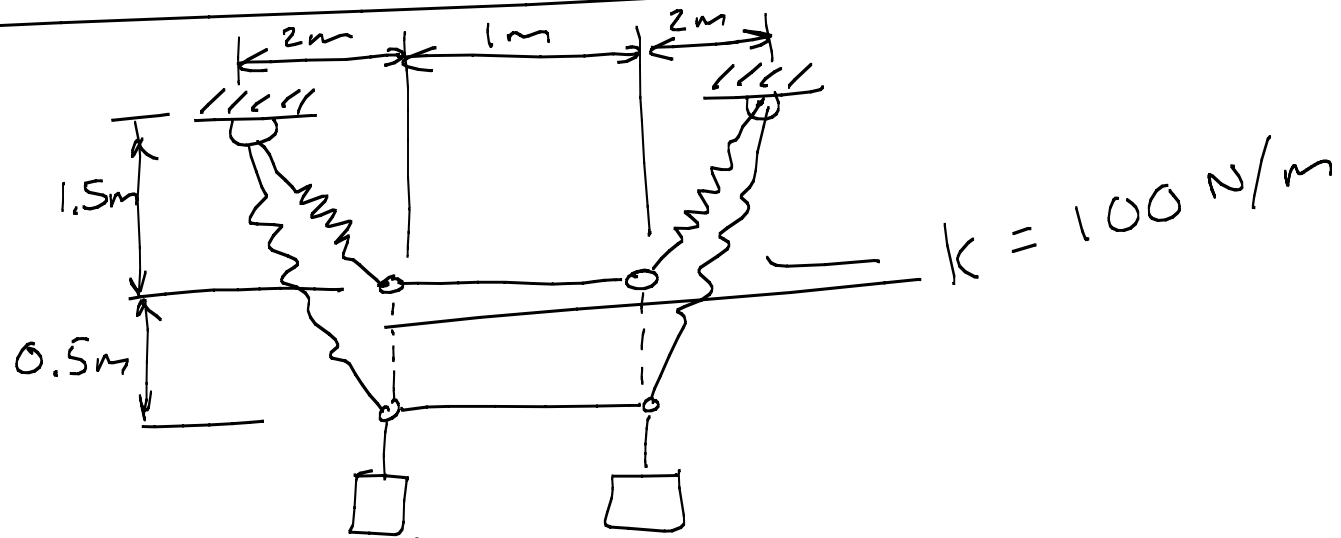
$$\uparrow \sum F_y = 0 \Rightarrow F_1 \sin 45^\circ - F_2 \sin 30^\circ = 0$$

2 Equations, 2 Unknowns

$$F_1 = 259 \text{ N}$$

$$F_2 = 366 \text{ N}$$

Example

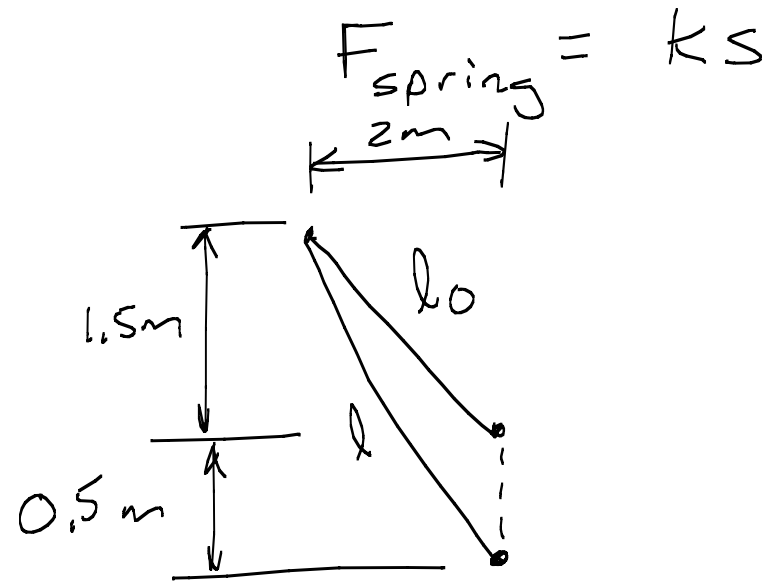
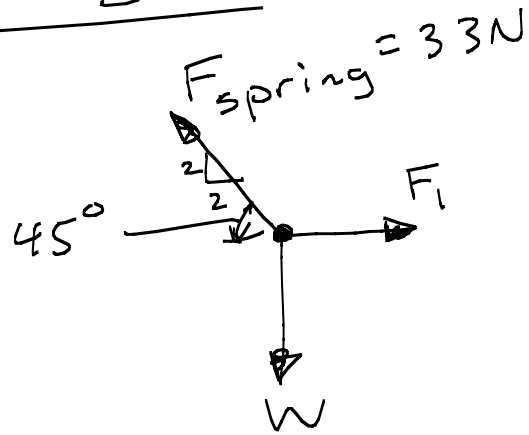


Determine

The mass of each object

same mass

FBD



$$l_0 = \sqrt{(2\text{m})^2 + (1.5\text{m})^2} = 2.5\text{m}$$

$$l = \sqrt{(2\text{m})^2 + (2\text{m})^2} = 2.83\text{m}$$

$$s = \Delta l = 2.83 - 2.5\text{m} = 0.33\text{m}$$

$$F_{\text{spring}} = (100\text{ N/m})(0.33\text{m}) = \underline{33\text{N}}$$

$$+\uparrow \sum F_y = 0 \Rightarrow (33\text{N})(\sin 45^\circ) - W = 0$$

$$W = 23.33\text{N}$$

$$m = \frac{W}{g} = \frac{23.33\text{N}}{9.81\text{m/s}^2} = \boxed{2.38\text{kg}}$$