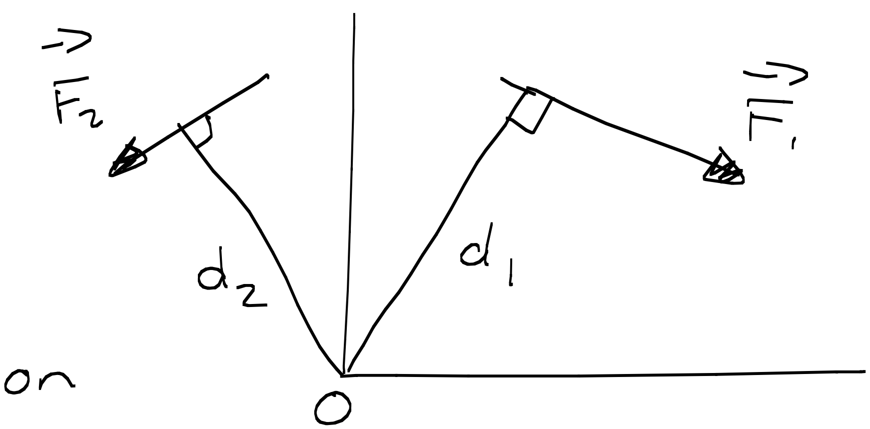
Resultant Moment in 2-D

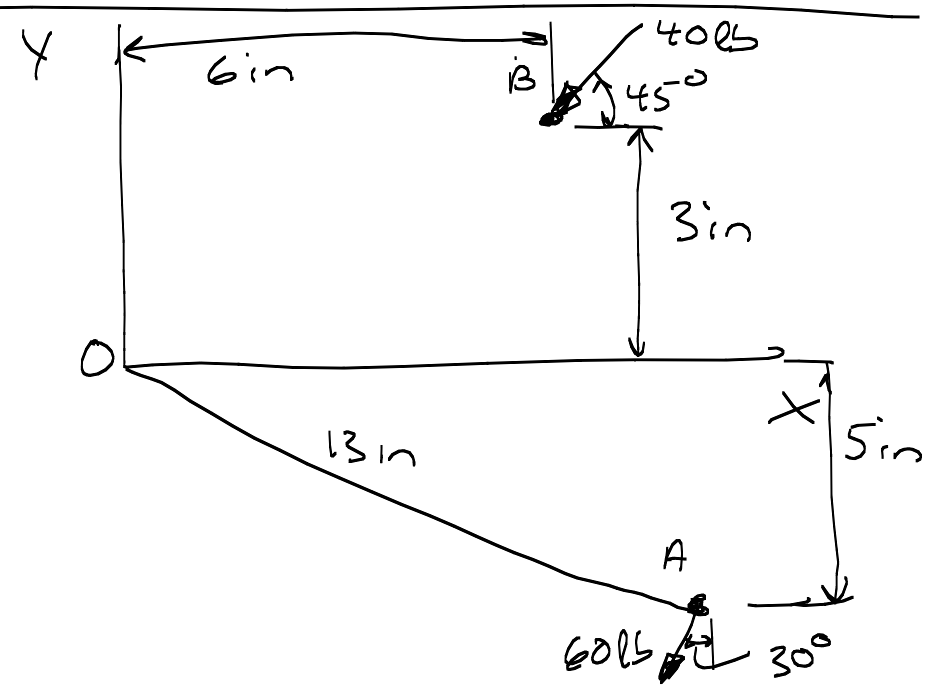


$$M_{R_O} = \sum Fd$$

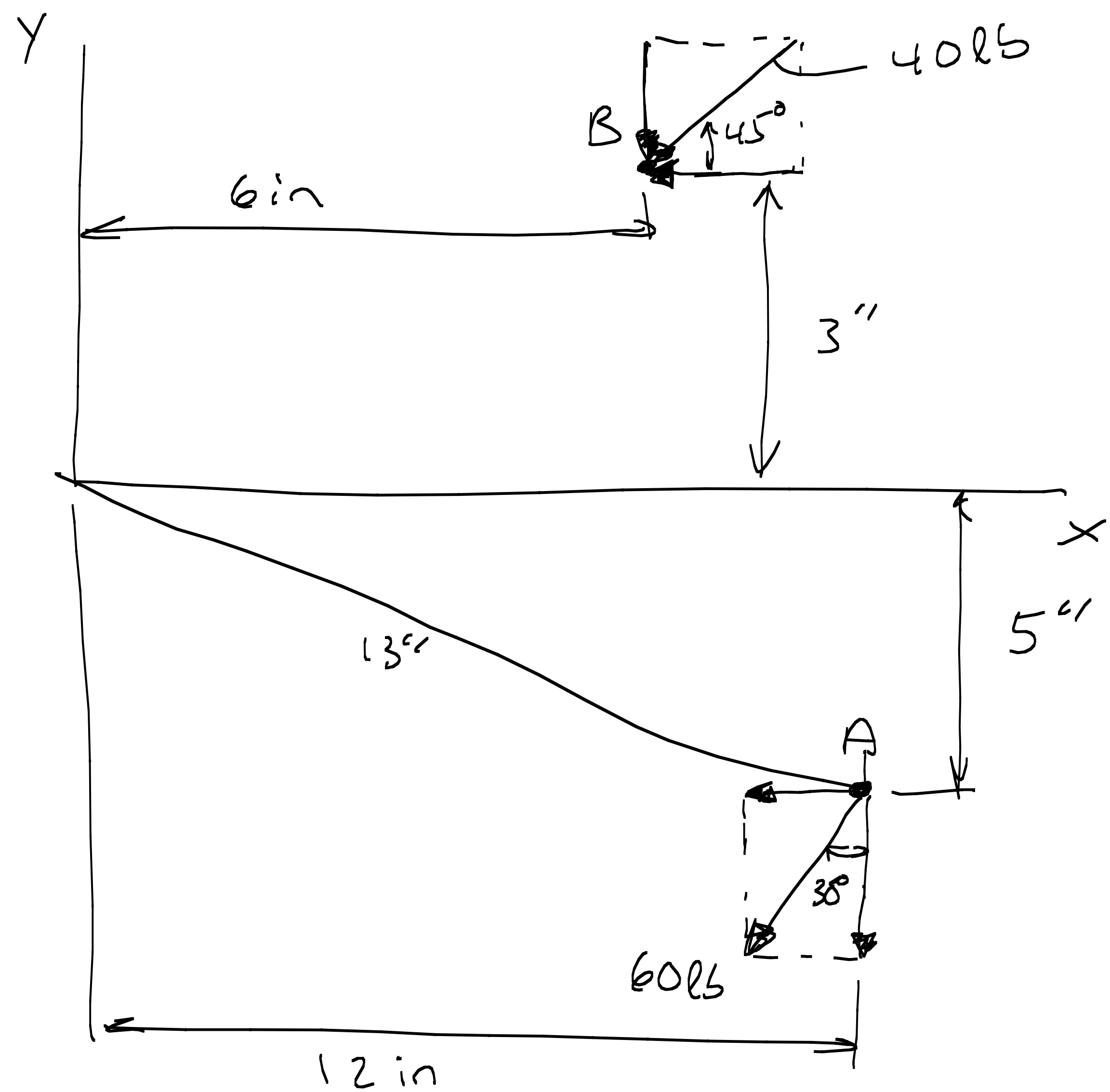
$$= -F_1 d_1 + F_2 d_2$$

(+ Indicates positive rotation in the equation

Example



Determine the resultant moment about point O



$$\curvearrowleft + \; \Sigma M_o = -(40\,lb \sin 45^\circ)(6\,in) + (40\,lb \cos 45^\circ)(3\,in)$$

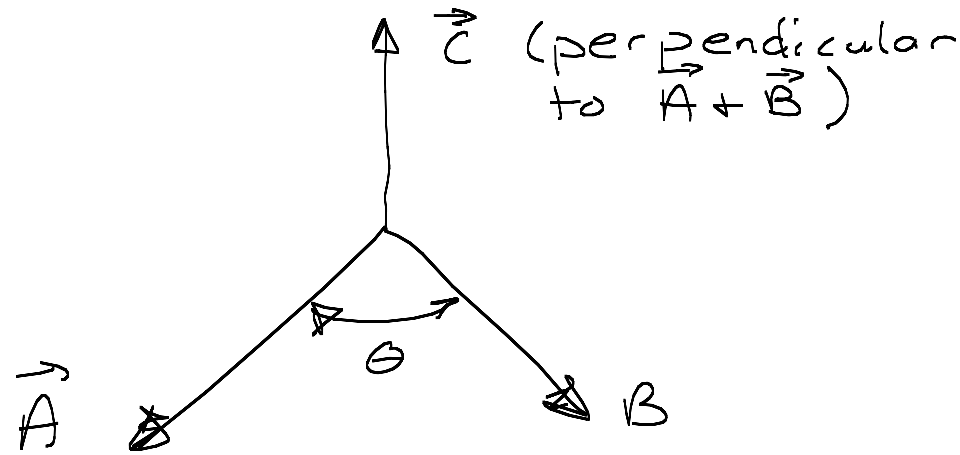
$$-(60\,lb\cos 30^\circ)(12\,in) - (60\,lb\sin 30^\circ)(5\,in)$$

$$\boxed{M_{RO} = -858\ lb\cdot in}$$

Cross Product

$$\vec{C} = \vec{A} \times \vec{B}$$

cross product



Magnitude

$$C = AB\sin\theta$$

Direction => Perpendicular to the plane containing $\vec{A}$ + $\vec{B}$

$$\vec{C} = \vec{A} \times \vec{B} = (AB\sin\theta)\,\vec{u}_c$$

Laws of Operation

$$\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A} \qquad (\vec{A} \times \vec{B}) = -(\vec{B} \times \vec{A})$$

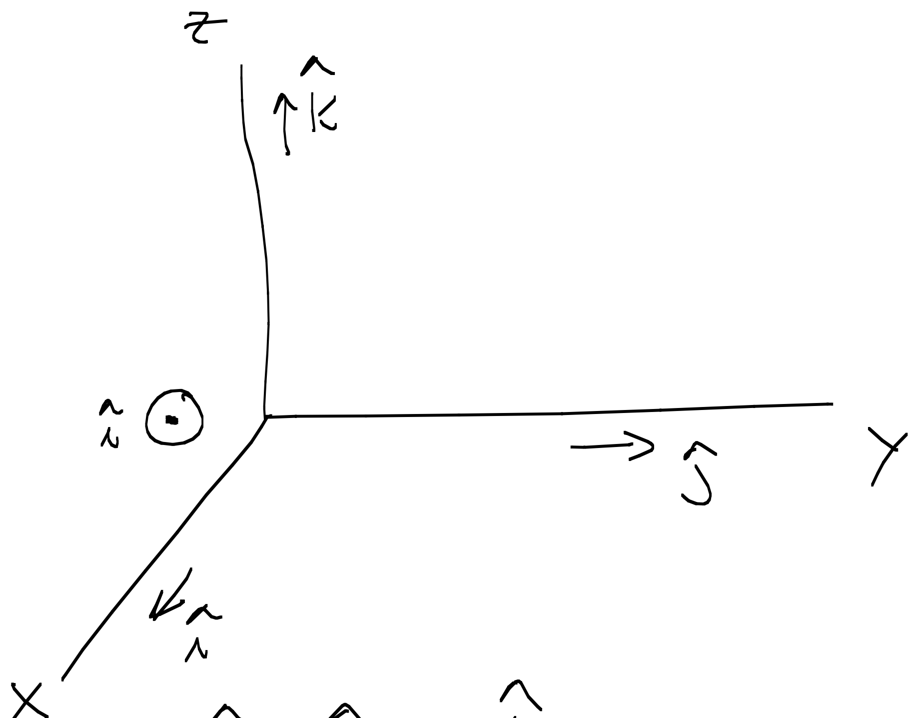
$$\underset{\text{scalar}}{a}(\vec{A} \times \vec{B}) = (a\vec{A}) \times \vec{B} = \vec{A} \times (a\vec{B})$$

$$\vec{A} \times (\vec{B} + \vec{D}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{D}$$

Cartesian Vectors

$$\vec{C} = (AB\sin\theta)\vec{u}_c$$

$$\left.\begin{aligned} \hat{i} \times \hat{i} &= 0 \\ \hat{j} \times \hat{j} &= 0 \\ \hat{k} \times \hat{k} &= 0 \end{aligned}\right\} \sin 0^\circ = 0$$



$$\hat{i} \times \hat{j} = [(1)(1)(\sin 90^\circ)]\,\hat{k} = \hat{k} \qquad \hat{j} \times \hat{i} = -\hat{k}$$

$$\hat{j} \times \hat{k} = \hat{i} \qquad \hat{k} \times \hat{j} = -\hat{i}$$

$$\hat{k} \times \hat{i} = \hat{j} \qquad \hat{i} \times \hat{k} = -\hat{j}$$

$$(\vec{A} \times \vec{B}) = \{A_x \hat{i} + A_y \hat{j} + A_z \hat{k}\} \times \{B_x \hat{i} + B_y \hat{j} + B_z \hat{k}\}$$

$$= \cancel{A_x B_x (\hat{i} \times \hat{i})} \to 0 + A_x B_y (\hat{i} \times \hat{j}) + A_x B_z (\hat{i} \times \hat{k})$$

$$+ A_y B_x (\hat{j} \times \hat{i}) + \cancel{A_y B_y (\hat{j} \times \hat{j})} \to 0 + A_y B_z (\hat{j} \times \hat{k})$$

$$+ A_z B_x (\hat{k} \times \hat{i}) + A_z B_y (\hat{k} \times \hat{j}) + \cancel{A_z B_z (\hat{k} \times \hat{k})} \to 0$$

$$= A_x B_y (\hat{k}) + A_x B_z (-\hat{j}) + A_y B_x (-\hat{k})$$

$$+ A_y B_z (\hat{i}) + A_z B_x (\hat{j}) + A_z B_y (-\hat{i})$$

$$\boxed{\vec{A} \times \vec{B} = (A_y B_z - A_z B_y)\hat{i} - (A_x B_z - A_z B_x)\hat{j} + (A_x B_y - A_y B_x)\hat{k}}$$

Can Express in Determinant Form

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

i-comp

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \hat{i}\left(A_y B_z - A_z B_y\right)$$

$\ominus$

$\hat{j}$-comp

$$\begin{array}{ccc} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & \ominus\; A_z \\ B_x & B_y & B_z \end{array}$$

for $\hat{j}$ only

$$-\hat{j}\left(A_x B_z - A_z B_x\right)$$

z-comp

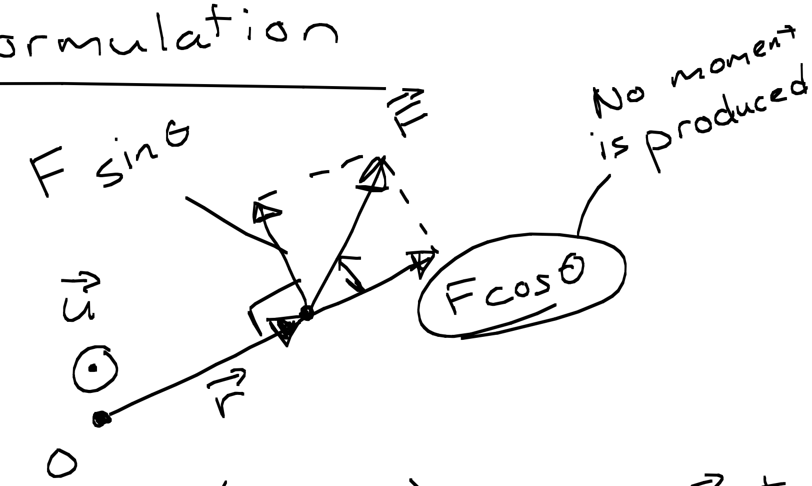
$$\begin{array}{ccc} \hat{i} & \hat{j} & \hat{k} \\ A_x & \ominus\; A_y & A_z \\ B_x & B_y & B_z \end{array}$$

$$\hat{k}\left(A_x B_y - A_y B_x\right)$$

$$\boxed{\vec{A} \times \vec{B} = \left(A_y B_z - A_z B_y\right)\hat{i} - \left(A_x B_z - A_z B_x\right)\hat{j} + \left(A_x B_y - A_y B_x\right)\hat{k}}$$

Moment of Force: Vector Formulation

$$\vec{M} = \vec{r} \times \vec{F}$$



$M_o = (F \sin\theta) r$ $\perp$ to $\vec{r}$ & $\vec{F}$

$M_o = F r \sin\theta$

$\vec{M}_o = (F r \sin\theta)\, \vec{u}$

$\vec{A} \times \vec{B} = (AB \sin\theta)\, \vec{u}$

$$\vec{M}_o = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

$$\boxed{\vec{M}_o = (r_y F_z - r_z F_y)\hat{i} - (r_x F_z - r_z F_x)\hat{j} + (r_x F_y - r_y F_x)\hat{k}}$$