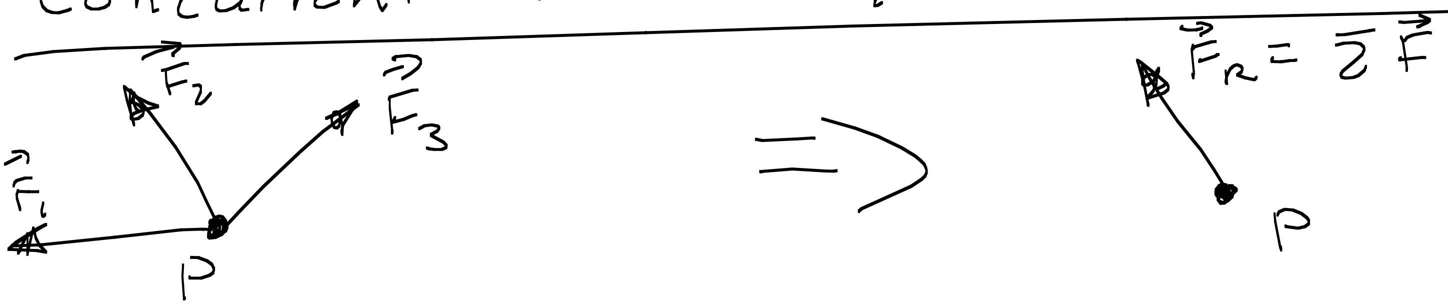
In General

$$\vec{F}_R = \Sigma \vec{F}$$

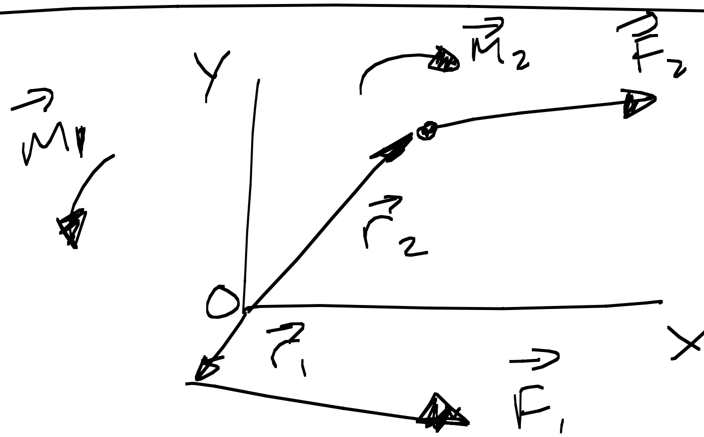
$$\vec{M}_{R_O} = \underbrace{\Sigma \vec{M}_c}_{\text{Existing Couple Moments}} + \underbrace{\Sigma \vec{M}_O}_{\text{Moments of Force about point O } (\vec{r} \times \vec{F})}$$

Simplification to a single resultant force

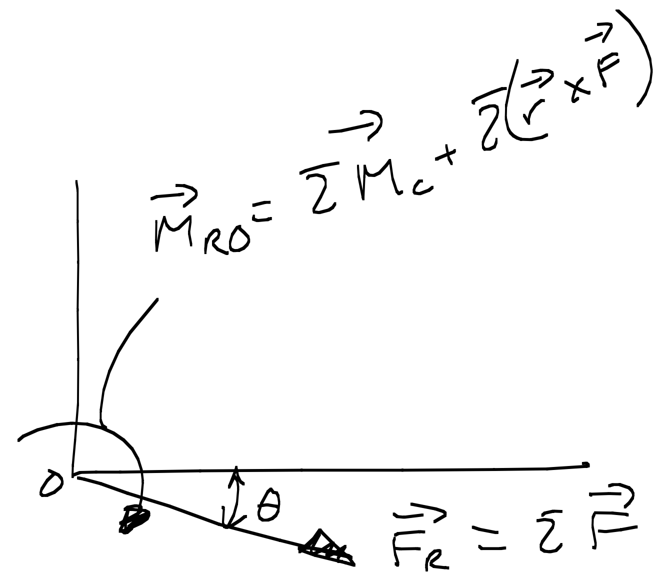
Concurrent Force System (Ch 2)



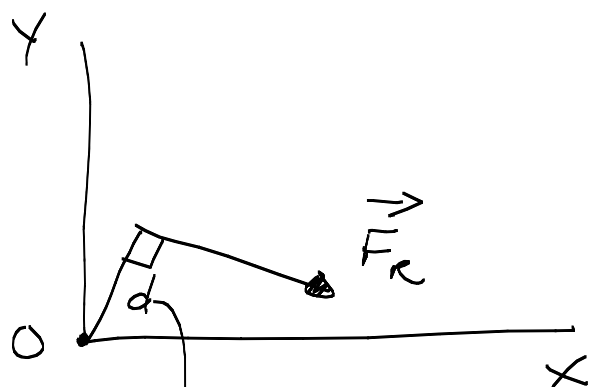
Coplanar Force Systems



⇒



⇒



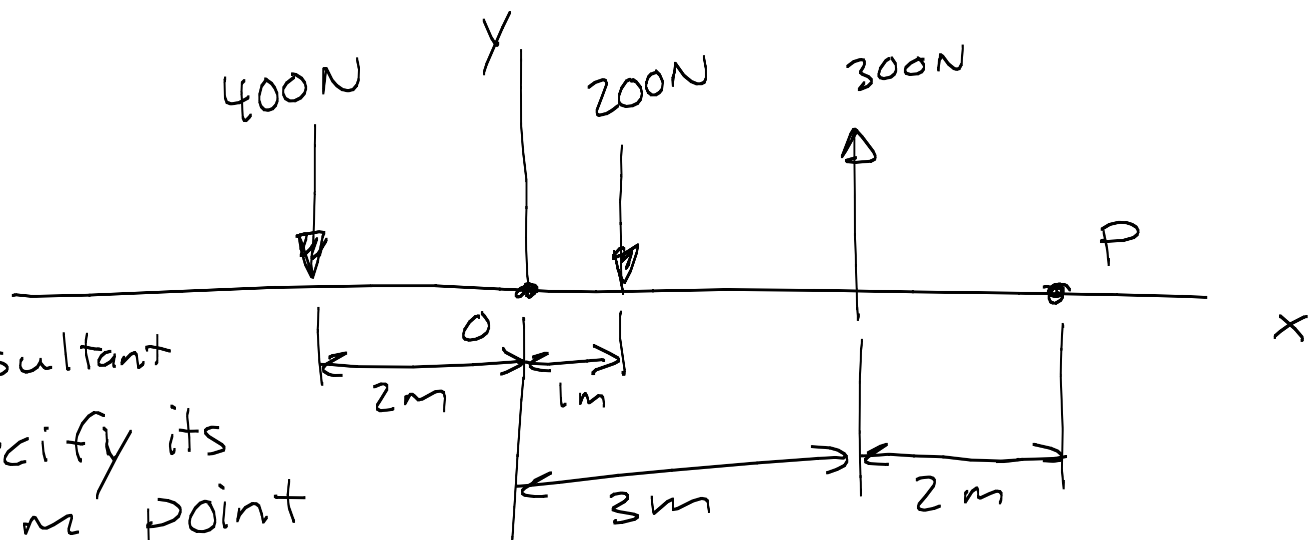
perpendicular distance, d, such that the moment produced by $\vec{F}_R$ about point O is equal to $\vec{M}_{RO}$

$$M_{RO} = F_R d$$

$$d = \frac{M_{RO}}{F_R}$$

Example

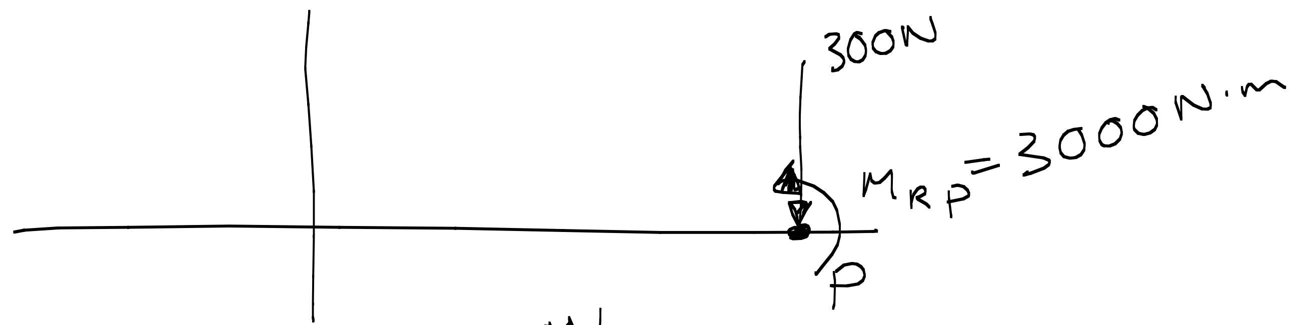
Replace the force system with an equivalent resultant force, and specify its location from point P.

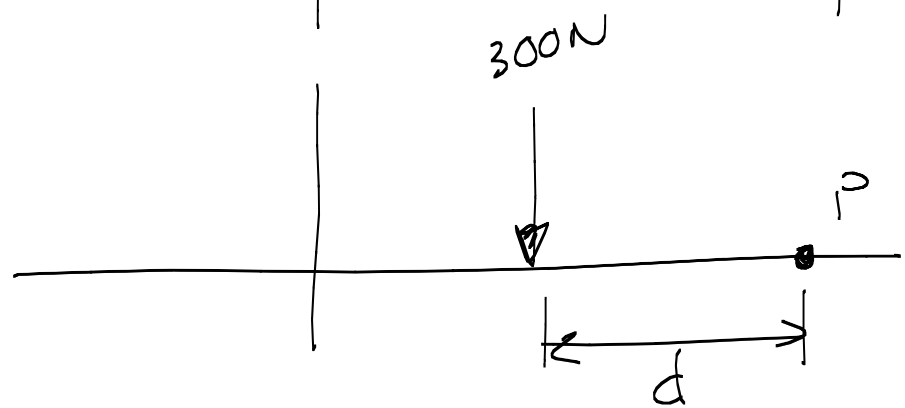


$$+\uparrow \Sigma F_y \Rightarrow F_R = -400N - 200N + 300N = \underline{-300N}$$

$+\circlearrowleft \sum M_P \Rightarrow M_{RP} = (400N)(2m) + (200N)(4m) - (300N)(2m)$

$M_{RP} = 3000\ N\cdot m$





$d = \frac{M_{RP}}{F_R}$

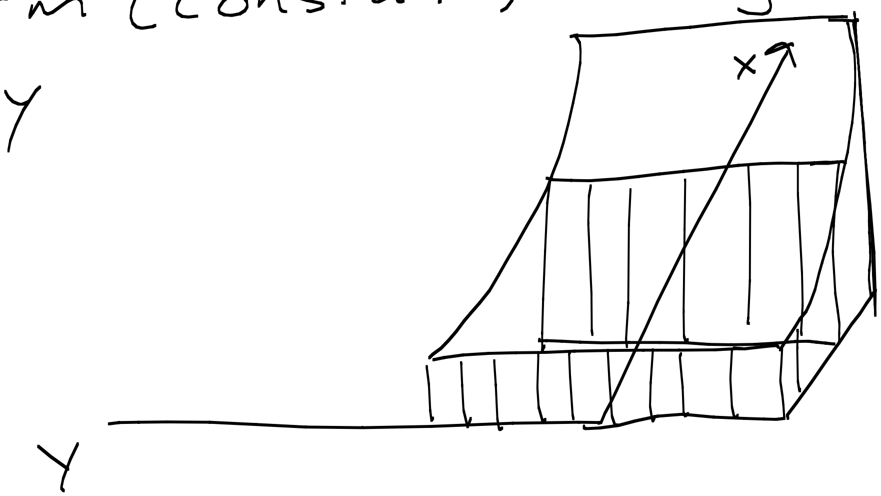
$= \frac{3000\ N\cdot m}{300N} \Rightarrow \boxed{d = 10m}$

Reduction of Distributed Loading

Distributed Loading: Load is distributed over the surface of a body

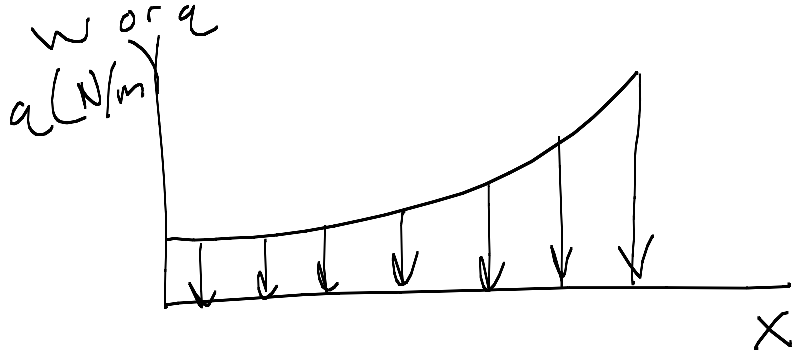
Examples => Wind, fluid, weight of a beam, etc

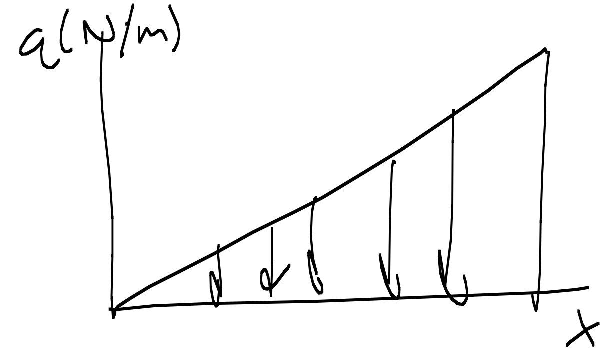
For now, we will assume that the distributed load is uniform (constant) along one axis along the body

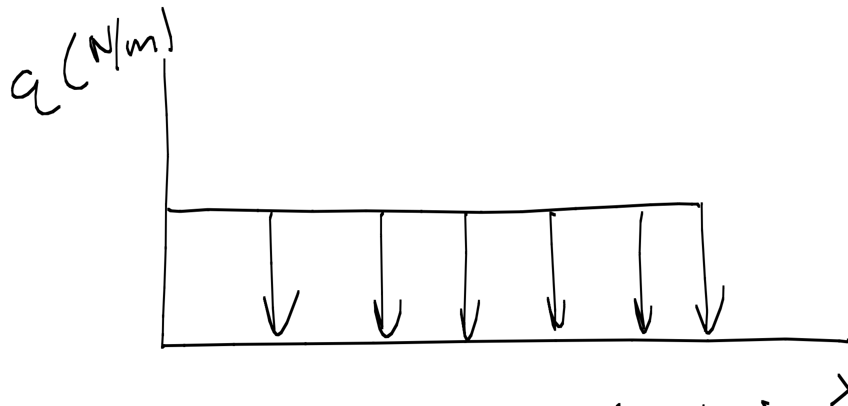


uniform along the y-axis

Load intensity diagram: Provides the distributions and direction of the distributed load







Goal => To replace the distributed load with an equivalent single resultant force system

Resultant Force => The area under the load intensity diagram

$$F_R = \int_L w(x)\,dx = \text{Area}$$

Location of the Resultant Force => The centroid of the load intensity diagram

$$\bar{x} = d = \frac{\overbrace{\int x\, w(x)\, dx}^{\frac{M_{RO}}{F_R} \;\; x \;\; dF}}{\int \underbrace{w(x)\, dx}_{dF}} = \frac{\int_A x\, dA}{\int_A dA}$$