Chapter 9: Center of Gravity and Centroid

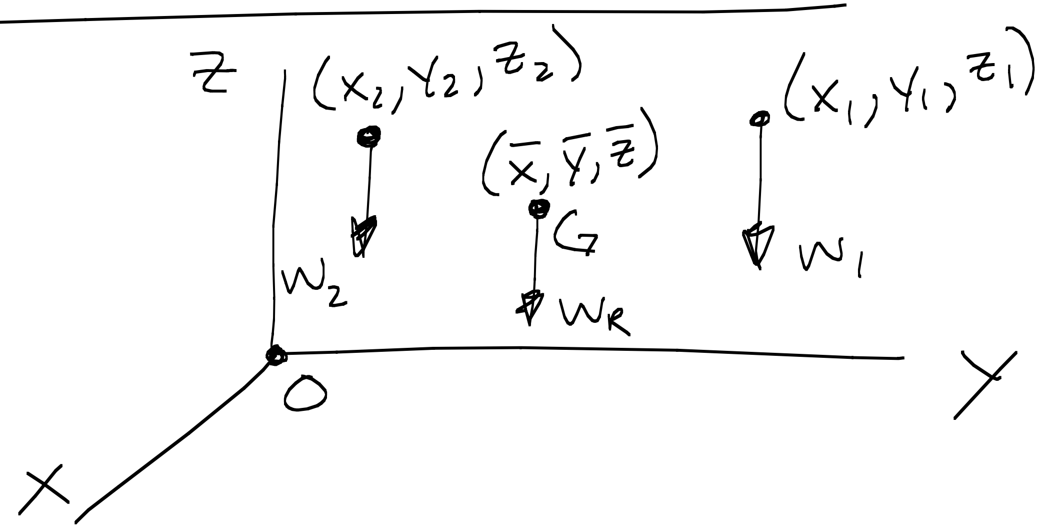
Center of Gravity: Locates the resultant weight of a system of particles

Series of Weights: Form a system of parallel forces

Replace the weights with an equivalent weight at a point of application

Determining the point of application

Sum moments



$$W_R = W_1 + W_2$$

$(\Sigma M_x)_o \Rightarrow$ $W_R \bar{y} = W_1 y_1 + W_2 y_2$

$$\boxed{\bar{y} = \frac{W_1 y_1 + W_2 y_2}{W_R}}$$

$(\Sigma M_y)_o \Rightarrow$ $W_R \bar{x} = W_1 x_1 + W_2 x_2$

$$\boxed{\bar{x} = \frac{W_1 x_1 + W_2 x_2}{W_R}}$$

In general

$$\bar{x} = \frac{\Sigma x W}{\Sigma W}, \quad \bar{y} = \frac{\Sigma y W}{\Sigma W}, \quad \bar{z} = \frac{\Sigma z W}{\Sigma W}$$

Center of an Area (Centroid)

$$\bar{X} = \frac{\Sigma x A}{\Sigma A}, \quad \bar{Y} = \frac{\Sigma y A}{\Sigma A}$$

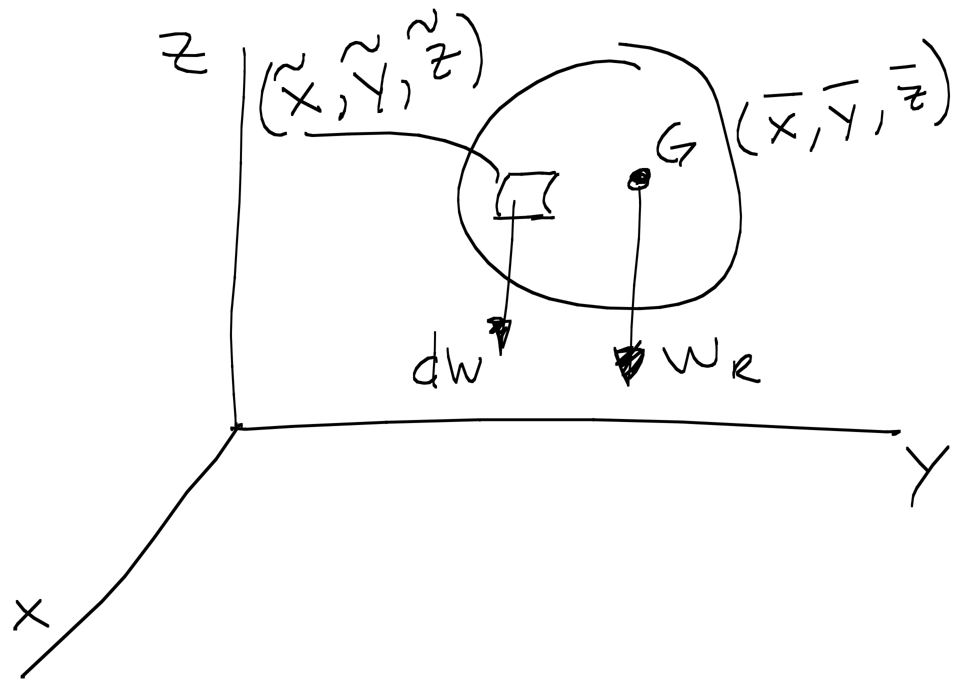
Center of Gravity of a Rigid Body

$\Sigma M_x \Rightarrow dM_x = \tilde{y}\, dW$

$\Sigma M_y = dM_y = \tilde{x}\, dW$

$$\bar{X} = \frac{\int \tilde{x}\, dW}{\int dW}$$

$$\bar{Y} = \frac{\int \tilde{y}\, dW}{\int dW}$$



Centroid

$$\bar{x} = \frac{\int \tilde{x}\, dA}{\int dA}, \quad \bar{y} = \frac{\int \tilde{y}\, dA}{\int dA}$$

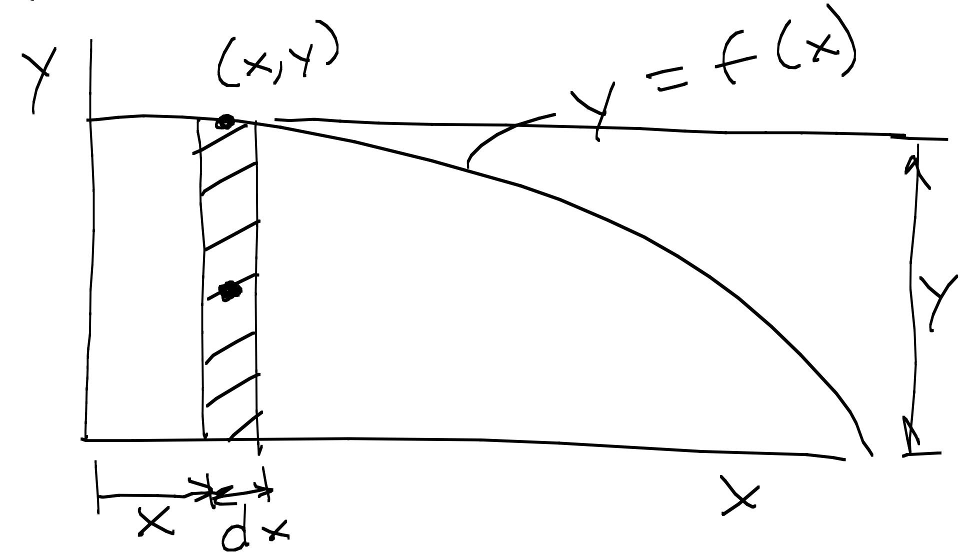
$\tilde{x}, \tilde{y}$ => Centroids of the differential element.

=> Double Integral

=> Alternative

Select a differential strip

$$\frac{\int \tilde{x}\, dA}{\int dA} \qquad \frac{\int \tilde{y}\, dA}{\int dA}$$



$\tilde{x} = x$

$\tilde{y} = y/2$

$dA = y\,dx$

$$\int \tilde{x}\, dA = \int xy\, dx$$

$$\int \tilde{y}\, dA = \int \frac{y}{2} y\, dx = \frac{1}{2}\int y^2 dx$$

$$\int dA = \int y\, dx$$

Use $y = f(x) \Rightarrow$ to solve for y in terms of x, and substitute

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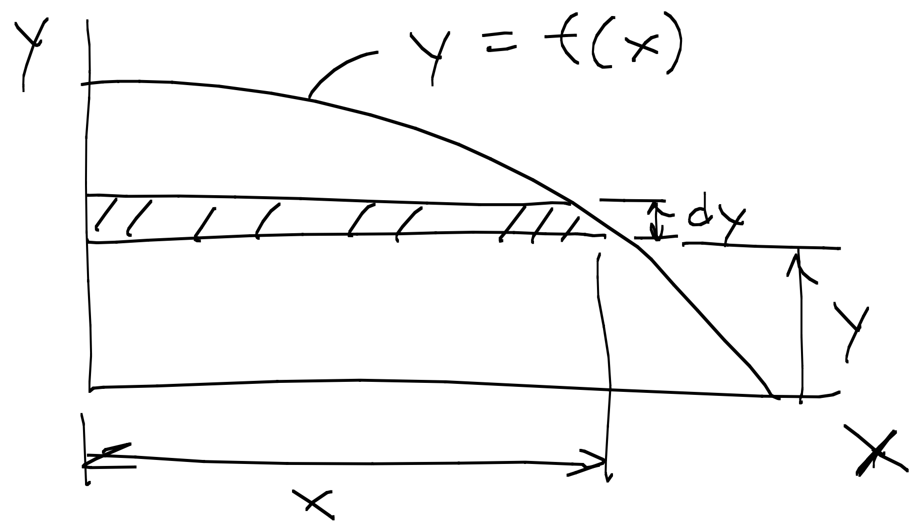
$\tilde{x} = x/2$

$\tilde{y} = y$

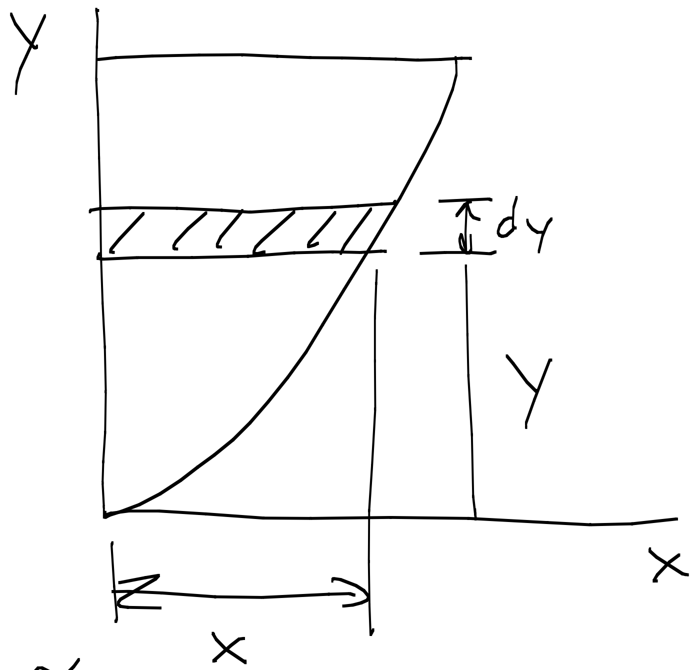
$dA = x\,dy$

$$\int \tilde{x}\, dA = \frac{1}{2}\int x(x\,dy) = \frac{1}{2}\int x^2 dy$$



$$\int \tilde{y}\, dA = \int y\, x\, dy$$

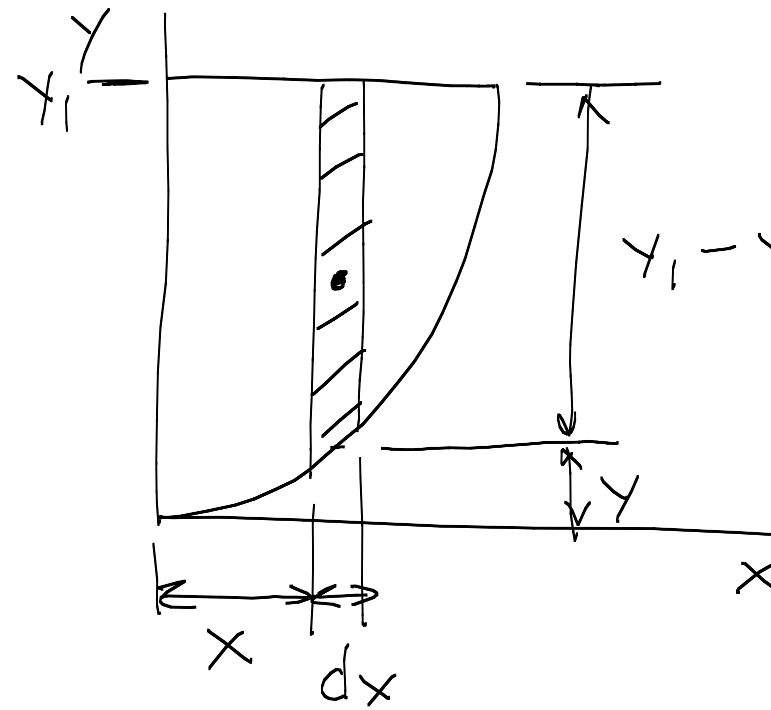
$$\int dA = \int x\, dy$$



$\tilde{x} = x/2$

$\tilde{y} = y$

$dA = x\, dy$



$\tilde{x} = x$

$\tilde{y} = y + \frac{y_1 - y}{2} = \frac{y}{2} + \frac{y_1}{2}$

$dA = (y_1 - y)(dx)$